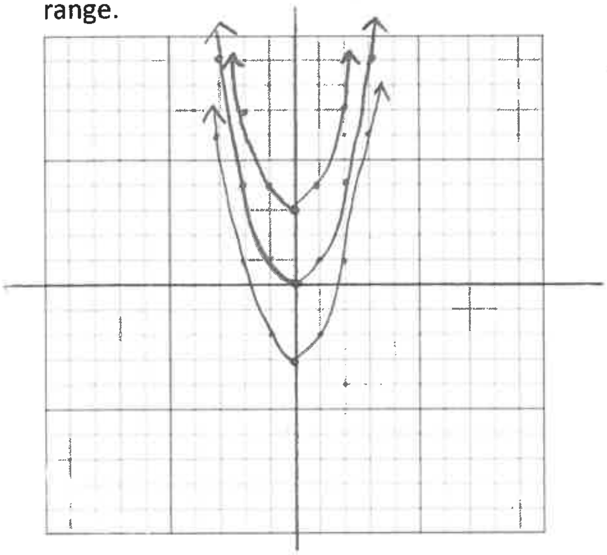
Name: Page 169 #4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17
Date:

3.1 – Transforming Graphs of Functions – Translations

Translation - to describe a function that moves an object a certain distance. The object is not altered in any other way.

Example 1: Graph the function $f(x) = x^2 + k$, let k = 0, 3, and – 3. Use a table of values and state the domain and range.



x	$y=x^2$
1	1
2	4
3	9

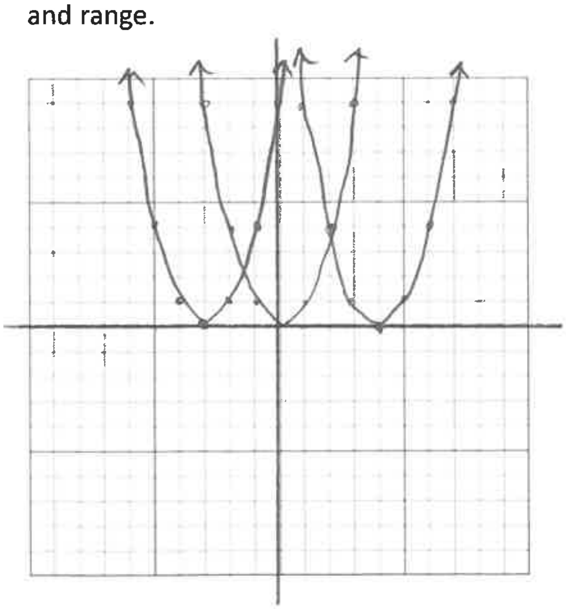
$y=x^2$
D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

$y = x^2 + 3$
D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \geq 3, y \in \mathbb{R}\}$

$y = x^2 - 3$
D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \geq -3, y \in \mathbb{R}\}$

Vertical Translation – The graph of $y - k = f(x)$ is a vertical translation of the graph of $y = f(x)$. The graph will be translated k units up or down. The point (x, y) on $y = f(x)$ corresponds to the point $(x, y + k)$ on $y - k = f(x)$.

Example 2: Graph the function $f(x) = (x - h)^2$, let h = 0, 4, and – 3. Use a table of values, and state the domain and range.



x	$y=x^2$
1	1
2	4
3	9

$(x-0)^2$
D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

$(x-4)^2$
D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

$(x+3)^2$
D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

Horizontal Translation – The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$. The graph will be translated h units left or right. The point (x, y) on $y = f(x)$ corresponds to the point $(x + h, y)$ on $y = f(x - h)$.

Pre-Calculus 30 – Unit 3 – Transforming Graphs of Functions
3.1 – Transforming Graphs of Functions

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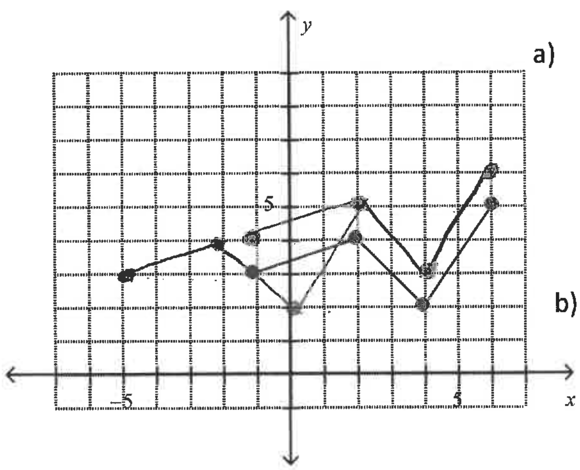
Lattice Point – a point at the intersection of two or more grid lines.

Graphing with Horizontal and/or Vertical Translations

1. Choose lattice points on the graph
2. Apply the translations to these points
3. Plot the translated point on the graph.
4. Sketch

Example 3: Here is the graph of $y = g(x)$. Sketch the image graph after each translation. Write the equation of the image graph In terms of the function g. State the domain and range of each function.



D: {x | −1 ≤ x ≤ 6, x ∈ ℝ}
R: {y | 2 ≤ y ≤ 5, y ∈ ℝ}

a) A translation of 4 units left

f(x) (x, y)	g(x) (x − 4, y)
(−1, 3)	(−5, 3)
(2, 4)	(−2, 4)
(4, 2)	(0, 2)
(6, 5)	(2, 5)

D: {x | −5 ≤ x ≤ 2, x ∈ ℝ}
R: {y | 2 ≤ y ≤ 5, y ∈ ℝ}

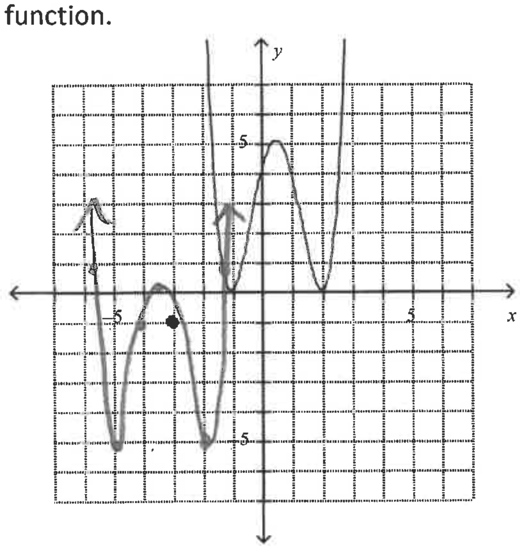
b) A translation of 1 unit up

f(x) (x, y)	g(x) (x, y + 1)
(−1, 3)	(−1, 4)
(2, 4)	(2, 5)
(4, 2)	(4, 3)
(6, 5)	(6, 6)

D: {x | −1 ≤ x ≤ 6, x ∈ ℝ}
R: {y | 3 ≤ y ≤ 6, y ∈ ℝ}

If you want another example of this go to Example 1 on page 165.

Example 4: Here is the graph of $y = f(x)$. Sketch the image graph after a translation of 4 units left and 5 units down. Write the equation of the image graph in terms of the function f. State the domain and range of each function.



D: {x | x ∈ ℝ}
R: {y | y ≥ 0, y ∈ ℝ}

f(x) (x, y)	g(x) (x − 4, y − 5)
(−1.8, 6)	(−5.8, 1)
(−1, 0)	(−5, −5)
(0, 4)	(−4, −1)
(0.5, 5.1)	(−3.5, 0.1)
(1, 4)	(−3, −1)
(2, 0)	(−2, −5)
(2.8, 6)	(−1.2, 1)

$y - k = f(x - h)$
up/down ↑ ↑ left/right

$y + 5 = f(x + 4)$

$h = -4$
$k = -5$

D: {x | x ∈ ℝ}
R: {y | y ≥ −5, y ∈ ℝ}

If you want another example of this go to example 2 on page 166.

Name: ______________________
Date: ______________________

Explicit Equation – an equation that is written in terms of the independent variable.

Example 5: The graph of $y = \frac{1}{x}$ is translated 3 units left and 2 units up. What is the equation of the image graph?

$$y - k = \frac{1}{x - h}$$

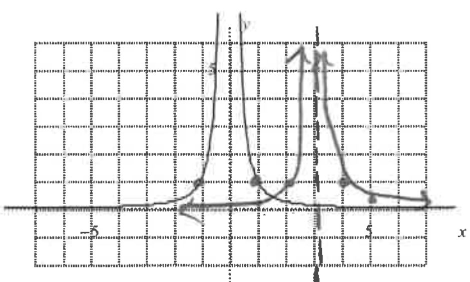
$$y - 2 = \frac{1}{x - 3}$$

If you want another example of this go to Example 3 on page 167.

Example 6: Describe how the graph of $y = \frac{1}{x^2}$ could have been translated to create the graph of each function below. What are the equations of the asymptotes of each image graph?

a) $y - 3 = \frac{1}{x^2}$

horizontal translation 3 (right)
No Vertical Translation
Asymptote on $f(x)$ is $x = 0$
Asymptote on $g(x)$ is $x = 0 + 3$
$= 3$



D: $\{x \mid x \neq 3, x \in \mathbb{R}\}$
R: $\{y \mid y > 0, y \in \mathbb{R}\}$

D: $\{x \mid x \neq 0, x \in \mathbb{R}\}$
R: $\{y \mid y > 0, y \in \mathbb{R}\}$

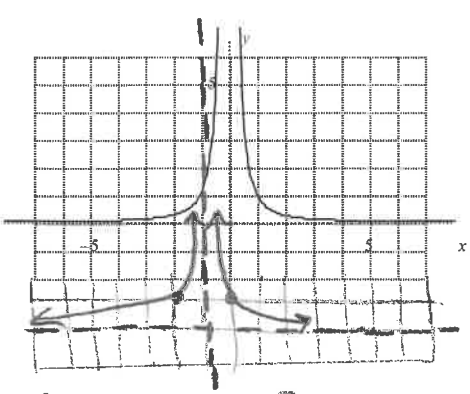
b) $y + 4 = \frac{1}{(x+1)^2}$

$h = -1$
$k = -4$

horizontal translation -1 (left)
Vertical translation -4 (down)

Vertical Asymptote $f(x)$ is $x = 0$
$g(x)$ is $x = 0 - 1$
$x = -1$

Horizontal Asymptote $f(x)$ $y = 0$
$g(x)$ $y = 0 - 4$
$y = -4$



D: $\{x \mid x \neq -1, x \in \mathbb{R}\}$
R: $\{y \mid y > -4, y \in \mathbb{R}\}$

D: $\{x \mid x \neq 0, x \in \mathbb{R}\}$
R: $\{y \mid y > 0, y \in \mathbb{R}\}$

If you want another example of this go to example 4 on page 167.

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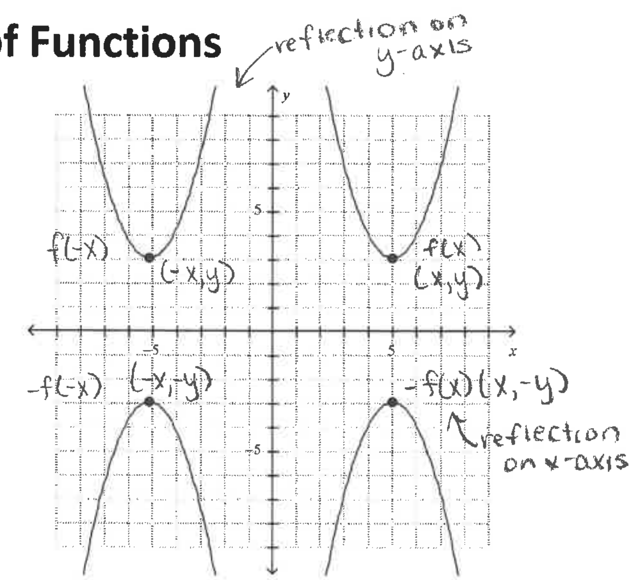
Date:

3.2 – Reflecting Graphs of Functions



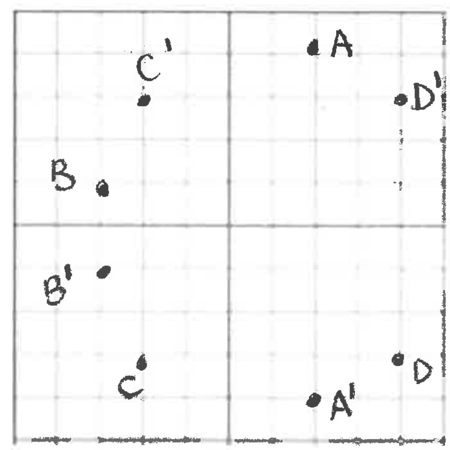
Reflection - a type of transformation in which the function is flipped across a line of reflection to create a new function. Each point of the function is the same distance from the reflection line as the original function is.

Reflecting in the x-axis – the graph of $y = -af(x)$ is the image of the graph $y = f(x)$ after a reflection in the x-axis. A point (x, y) on $y = f(x)$ would correspond to the point $(x, -y)$ on $y = -af(x)$.

Reflecting in the y-axis – the graph of $y = f(-x)$is the image of the graph of $y = f(x)$ after a reflection in the y-axis. The point (x, y) on $y = f(x)$ would correspond to the point $(-x, y)$ on $y = f(-x)$.

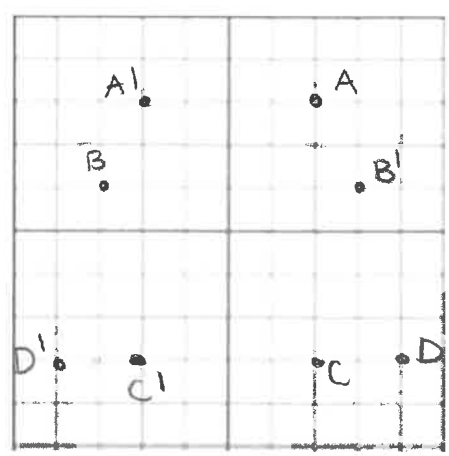
Example 1: For each of the following points label which quadrant they are in, plot the point on the graph, and write the reflected point across the x-axis.

Point	Quadrant	Point Reflected on x-axis	Quadrant
A (2, 4)	I	2, -4	IV
B (-3, 1)	II	-3, -1	III
C (-2, -3)	III	-2, 3	II
D (4, -3)	IV	4, -3	I



Example 2: For each of the following points label which quadrant they are in, plot the point on the graph, and write the reflected point across the y-axis.

Point	Quadrant	Point Reflected on x-axis	Quadrant
A (2, 4)	I	(-2, 4)	II
B (-3, 1)	II	(3, 1)	I
C (-2, -3)	III	(2, -3)	IV
D (4, -3)	IV	(-4, -3)	III



Name: ______________________
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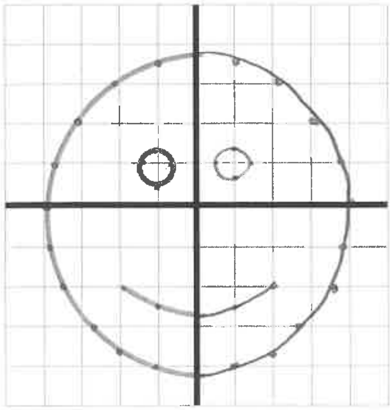
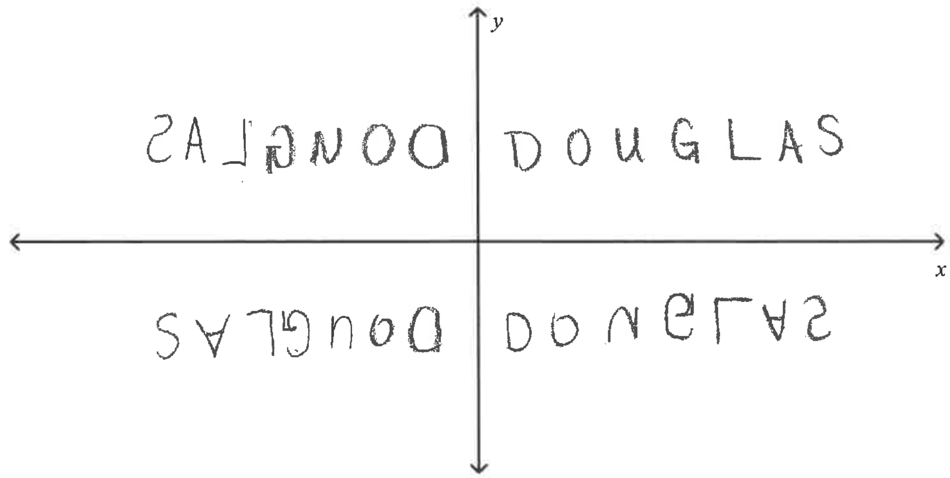
Example 3: Reflect the following image across the y-axis., then write your name reflected in each quadrant.





Sketching the Graph of a Polynomial Functions after a Reflection

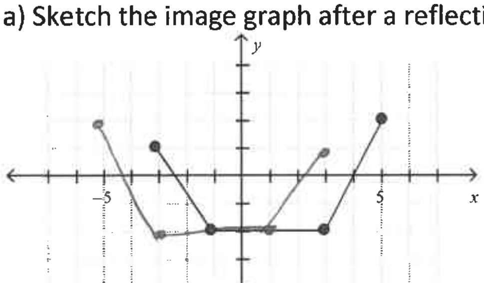
1. Determine lattice points for the graph or estimate coordinates
2. Reflect the points remembering the following:

$$y = f(x) \rightarrow (x, y)$$
$$y = -f(x) \rightarrow (x, -y)$$
$$y = f(-x) \rightarrow (-x, y)$$
$$y = -f(-x) \rightarrow (-x, -y)$$

3. Plot the points then sketch.

Example 4: Here is the graph of y = g(x).

a) Sketch the image graph after a reflection in the y-axis. State the domain and range of each function.

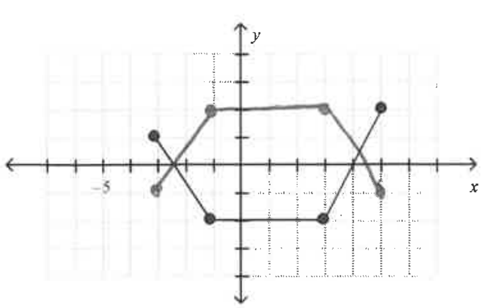


D: $\{x \mid -3 \le x \le 5, x \in \mathbb{R}\}$
R: $\{y \mid -2 \le y \le 2, y \in \mathbb{R}\}$

g(x) (x,y)	g(-x) (-x,y)
(-3, 1)	(3, 1)
(-1, -2)	(1, -2)
(3, -2)	(-3, -2)
(5, 2)	(-5, 2)

D: $\{x \mid -5 \le x \le 3, x \in \mathbb{R}\}$
R: $\{y \mid -2 \le y \le 2, y \in \mathbb{R}\}$

b) Sketch the image graph after a reflection in the x-axis. State the domain and range of each function.



g(x) (x,y)	-g(x) (x,-y)
(-3, 1)	(-3, -1)
(-1, -2)	(-1, 2)
(3, -2)	(3, 2)
(5, 2)	(5, -2)

D: $\{x \mid -3 \le x \le 5, x \in \mathbb{R}\}$
R: $\{y \mid -2 \le y \le 2, y \in \mathbb{R}\}$

D: $\{x \mid -3 \le x \le 5, x \in \mathbb{R}\}$
R: $\{y \mid -2 \le y \le 2, y \in \mathbb{R}\}$ *If you want another example of this go to example 1 on page 180.*

Name: ______________________
Date: ______________________

Writing an Equation of a Reflection Image

1. Determine if you are reflecting in the x-axis or the y-axis
2. Substitute your x or your y value in as a – 1
3. Simplify

Example 5: The graph of $y = \frac{1}{-2x^2-0.5}$ was reflected in the x-axis and on the y-axis. What is an equation for each of the images?

$f(-x)$ ★reflected on y-axis

$y = \frac{1}{-2(-x)^2-0.5}$

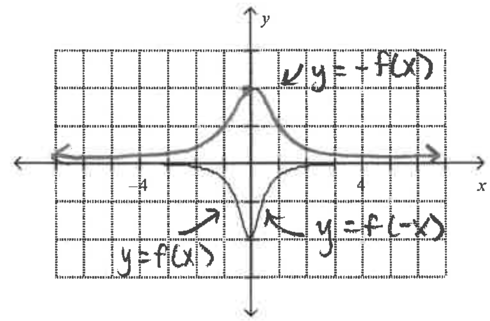
$y = \frac{1}{-2x^2-0.5}$

★Doesn't change

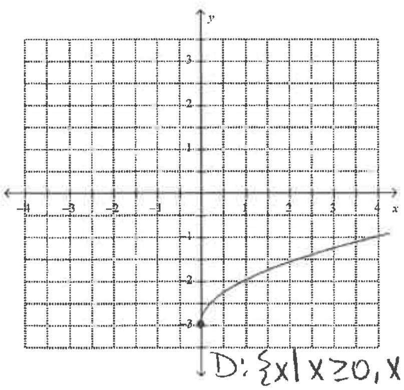
$-f(x)$ ★reflected in x-axis

$y = -\left(\frac{1}{-2x^2-0.5}\right)$

$y = \frac{1}{2x^2+0.5}$

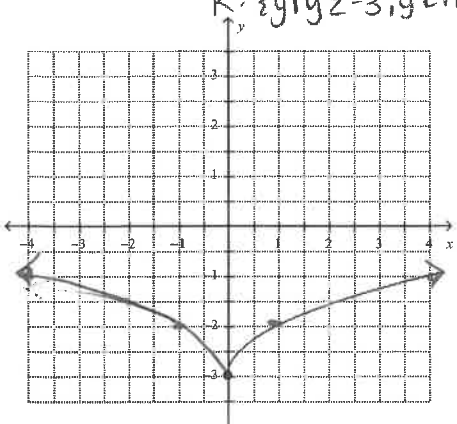


Example 6: Here is the graph of $y = \sqrt{x} - 3$. Fill in a table of values to show the reflection of $y = -f(x)$, $y = f(-x)$ and $y = -f(-x)$. Sketch each function, find the domain and range of each function, and determine an equation for each situation.

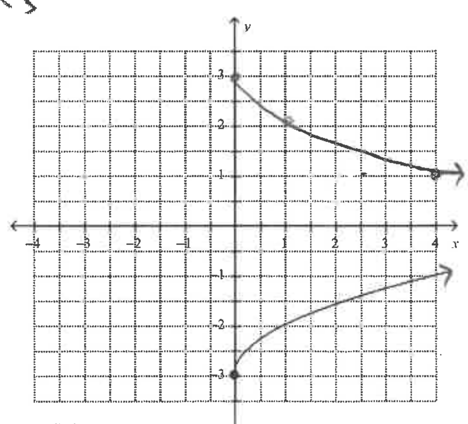


D: $\{x \mid x \geq 0, x \in \mathbb{R}\}$
R: $\{y \mid y \geq -3, y \in \mathbb{R}\}$

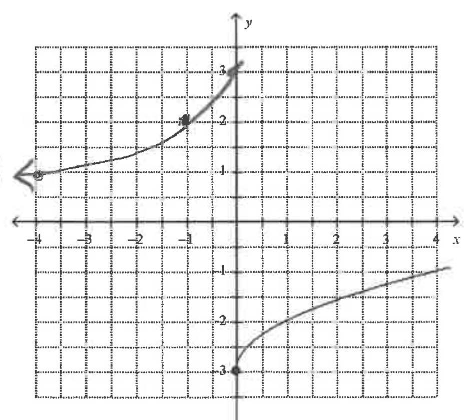
$f(x)$ (x, y)	$f(-x)$ $(-x, y)$ (y axis)	$-f(x)$ $(x, -y)$ (x axis)	$-f(-x)$ $(-x, -y)$
(0, 3)	(0, -3)	(0, 3)	(0, 3)
(1, -2)	(-1, -2)	(1, 2)	(-1, 2)
(4, -1)	(-4, -1)	(4, 1)	(-4, 1)



$f(-x)$
D: $\{x \mid x \leq 0, x \in \mathbb{R}\}$
R: $\{y \mid y \geq -3, y \in \mathbb{R}\}$



$-f(x)$
D: $\{x \mid x \geq 0, x \in \mathbb{R}\}$
R: $\{y \mid y \leq 3, y \in \mathbb{R}\}$



$-f(-x)$
D: $\{x \mid x \leq 0, x \in \mathbb{R}\}$
R: $\{y \mid y \leq 3, y \in \mathbb{R}\}$

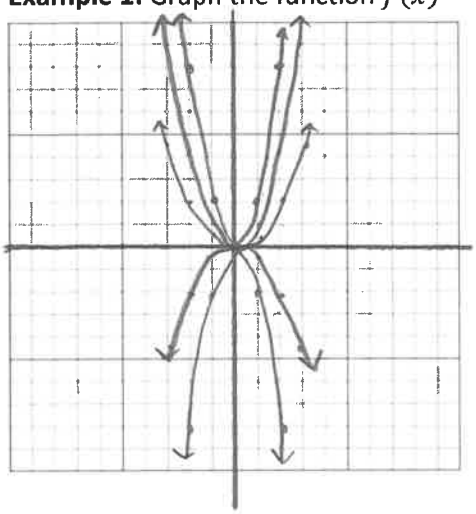
If you want another example of this go to Example 2 on page 181.

Name: Page 201 # 3,4,5,7, 8,9,11,13
Date:

3.3 – Stretching and Compressing Graphs of Functions

Vertical Stretch, Compressions or Reflections – When the graph of $y = af(x)$ is the image of the graph of $y = f(x)$. The point (x, y) on $y = f(x)$ corresponds to the point (x, ay) on $y = af(x)$.

Example 1: Graph the function $f(x) = ax^2$, let a = 1, 2, 0.5, – 2 and – 0.5 . Use a table of values.



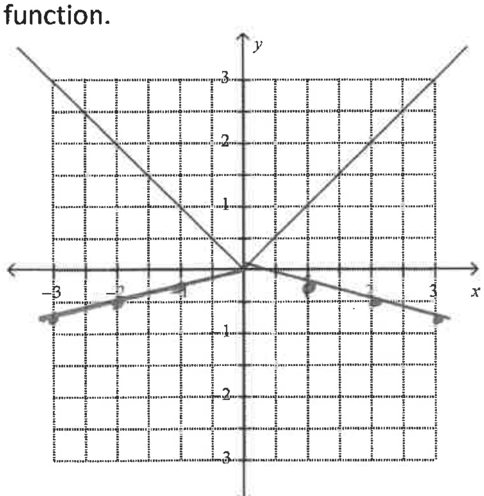
x	$y=x^2$	$y=2x^2$	$y=0.5x^2$	$y=-2x^2$	$y=-0.5x^2$
1	1	2	0.5	-2	-0.5
2	4	8	2	-8	-2
3	9	18	4.5	-18	-4.5

Vertical Stretch – In the graph $y = af(x)$, if $|a| > 1$, then the graph of $y = f(x)$ has been vertically stretched by a factor of $|a|$.
Vertical Compression – In the graph $y = af(x)$, if $0 < |a| < 1$, then the graph of $y = f(x)$ has been vertically compressed by a factor of $|a|$.
Vertical Reflection – In the graph $y = af(x)$, if $a < 0$, the graph of $y = f(x)$ has a reflection in the x-axis as well as a possible stretch or compression.

Sketching the Graph of a Function with a Vertical Stretch and Reflection

1. Choose lattice points on the graph
2. Apply the transformation by the factor of a
3. Plot the new points and sketch

Example 2: Here is the graph of $y = f(x)$. Sketch the graph of $y = -\frac{1}{4}f(x)$. State the domain and range of each function.



$f(x)$ (x,y)	$g(x)$ $(x, -\frac{1}{4}y)$
(1,1)	(1,-1/4)
(2,2)	(2,-1/2)
(3,3)	(3,-3/4)

$a = -1/4$

$y = -\frac{1}{4}f(x)$

D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \le 0, y \in \mathbb{R}\}$

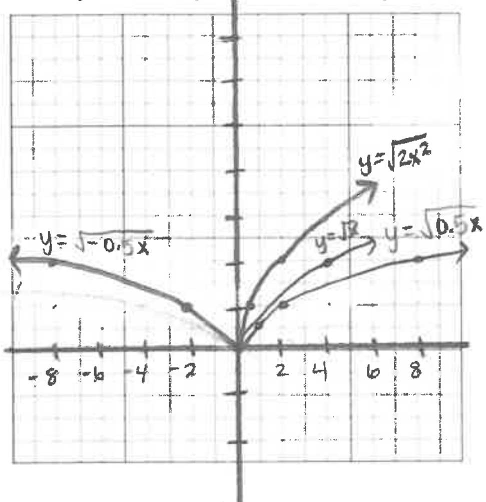
D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \ge 0, y \in \mathbb{R}\}$

If you want another example of this go to example 1 on page 196.

Name: ____________________
Date: ____________________

Horizontal Stretch, Compressions or Reflections – When the graph of $y = bf(x)$ is the image of the graph of $y = f(x)$. The point (x, y) on $y = f(x)$ corresponds to the point $\left(\frac{x}{b}, y\right)$ on $y = f(bx)$.

Example 3: Graph the function $f(x) = \sqrt{bx}$, let a = 1, 2, 0.5, and – 0.5 . Use a table of values.



$f(x)=\sqrt{x}$

x	y
1	1
2	$\sqrt{2}$
4	2

$f(x)=\sqrt{2x}$

x	y
1	$\sqrt{2(1)}=\sqrt{2}$
2	$\sqrt{2(2)}=2$
0.5	$\sqrt{2(0.5)}=1$

$f(x)=\sqrt{0.5x}$

x	y
2	$\sqrt{0.5(2)}=1$
8	$\sqrt{0.5(8)}=2$

$f(x)=\sqrt{-0.5x}$

x	y
-2	1
-8	2

$f(x)=\sqrt{x}$	$f(x)=\sqrt{2x}$	$f(x)=\sqrt{0.5x}$	$f(x)=\sqrt{-0.5x}$	$y=\sqrt{bx}$
(1,1)	(0.5,1)	(2,1)	(-2,-1)	$\left(\frac{x}{b}, y\right)$
(4,2)	(2,2)	(8,2)	(-8,2)	
(x,y)	$\left(\frac{x}{2}, y\right)$	(2x,y)	(-2x,y)	

Horizontal Stretch – In the graph $y = f(bx)$, if $0 < |b| < 1$, then the graph of $y = f(x)$ has been horizontally stretched by a factor of $\frac{1}{|b|}$.

Horizontal Compression – In the graph $y = f(bx)$, if $|b| > 1$, then the graph of $y = f(x)$ has been horizontally compressed by a factor of $\frac{1}{|b|}$.

Horizontal Reflection – In the graph $y = f(bx)$, if $b < 0$, the graph of $y = f(x)$ has a reflection in the y-axis as well as a possible stretch or compression.

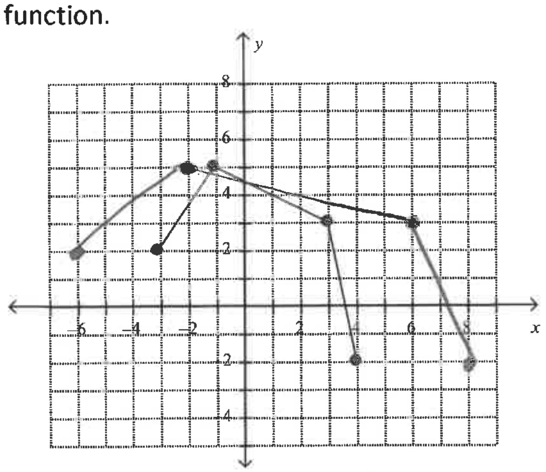
Sketching the Graph of a Function with a Horizontal Stretch and Reflection

1. Choose lattice points on the graph
2. Apply the transformation by the factor of $\frac{1}{b}$
3. Plot the new points and sketch

Example 4: Here is the graph of $y = g(x)$. Sketch the graph of $y = g(0.5x)$. State the domain and range of each function.

$b = 0.5 = 1/2$

$\frac{x}{b} = \frac{x}{\frac{1}{2}} = 2x$



g(x) (x,y)	g(0.5x) (2x,y)
(-3,2)	(-6,2)
(-1,5)	(-2,5)
(3,3)	(6,3)
(4,-2)	(8,-2)

D: $\{x \mid -3 \le x \le 4, x \in \mathbb{R}\}$
R: $\{y \mid -2 \le y \le 5, y \in \mathbb{R}\}$

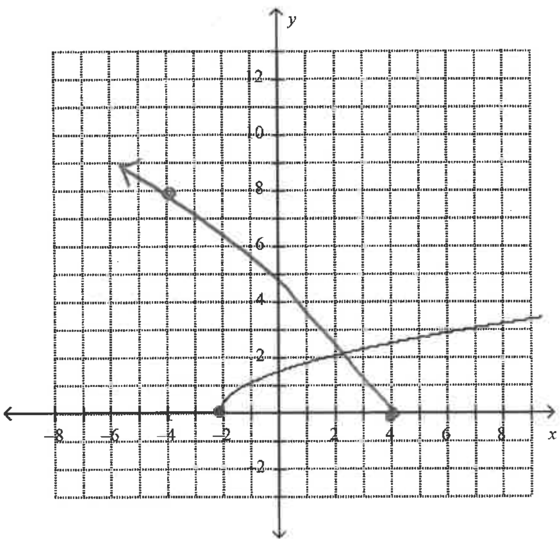
D: $\{x \mid -6 \le x \le 4, x \in \mathbb{R}\}$
R: $\{y \mid -2 \le y \le 5, y \in \mathbb{R}\}$

If you want another example of this go to example 2 on page 198.

Name: ______________________
Date: ______________________

Stretch, Compression and Reflection – The point (x, y) on $y = f(x)$ corresponds to the point $\left(\frac{x}{b}, ay\right)$ on $y = af(bx)$.

Example 5: Here is the graph of $y = f(x)$. Sketch the graph of $y = 4f(-0.5x)$. State the domain and range.



$f(x)$ (x,y)	$\left(\frac{x}{-0.5}, 4y\right)$
$(-2,0)$	$(4,0)$
$(2,2)$	$(-4,8)$

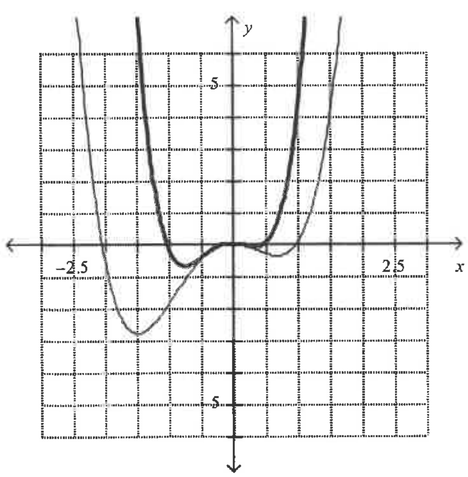
$D: \{x \mid x \le -4, x \in \mathbb{R}\}$
$R: \{y \mid y \ge 0, y \in \mathbb{R}\}$

If you want another example of this go to example 3 on page 199.

Determining an Equation After a Transformation

1. Identify corresponding points on both functions, such as intercepts or local maximum/minimums
2. Determine the values of a and b necessary to go from (x, y) to $\left(\frac{x}{b}, ay\right)$
3. Substitute into $y = af(bx)$.

Example 6: The graphs of $y = f(x)$ and its image after a vertical and/or horizontal compression are shown. Write an equation of the image graph in terms of the function f.



$f(x)$ (x,y)	$g(x)$ $\left(\frac{x}{b}, ay\right)$
$\boxed{(-2,0)}$	$\boxed{(-1,0)}$
$(1,0)$	$(0.5,0)$
$\boxed{(-1.5,-2.8)}$	$\boxed{(-0.75,-0.75)}$

$\frac{x}{b} = x'$ $\qquad$ $ay = y'$

$\frac{-2}{b} = -1$ $\qquad$ $\frac{a(-2.8)}{-2.8} = \frac{-0.75}{-2.8}$

$\frac{-2}{-1} = b$ $\qquad$ $a = \frac{0.75}{2.8}$

$2 = b$

$y = \frac{0.75}{2.8} f(2x)$

If you want another example of this go to example 4 on page 200.

Name: Page 226 # 3, 4, 5, 6, 7
Date: ____________

3.4 – Combining Transformations of Functions – Part 1

Combining Transformations = $y - k = af(b(x-h))$ is the image of the graph of $y = f(x)$ after the transformations:

- Horizontal stretch or compression by the factor of $\frac{1}{|b|}$
- Reflection on the y-axis if $b < 0$
- Vertical stretch or compression by the factor of $|a|$
- Reflection on the x-axis if $a < 0$
- Horizontal translation of h units
- Vertical translation of k units.

Point (x, y) on the graph y = f(x) corresponds to the point $\left(\frac{x}{b} + h,\ ay + k\right)$ on the graph $y - k = af(b(x-h))$.

Example 1: a) Describe all the transformations for the following function: $y - 6 = f(4(x+2))$

horizontal compression of 1/4
vertical translation 6 (up)
horizontal translation −2 (left)

$a = 1$
$b = 4$
$h = -2$
$K = 6$

b) The point (2,4) is on the graph y = f(x), what point will it be on the transformed function?

$\left(\frac{x}{4} - 2, y + 6\right)$

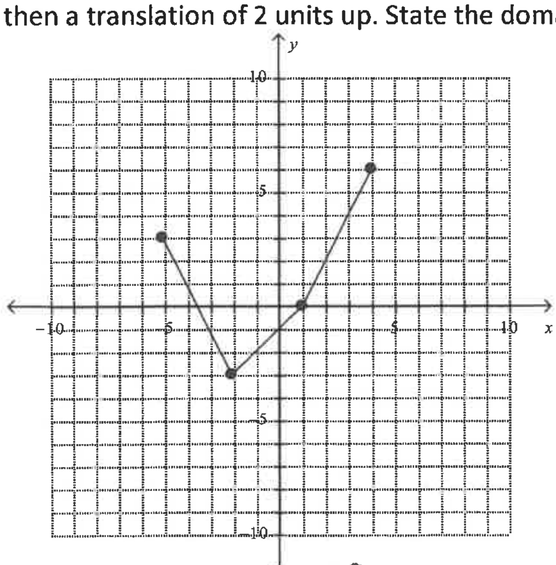
$\frac{2}{4} - 2, \ 4 + 6$

$(-1.5, 10)$

Sketching Transformations of Graphs

1. Choose lattice points
2. Determine all transformations
3. Apply all stretches and compressions first
4. Apply all reflections
5. Apply all translations

Example 2: Here is the graph of $y = g(x)$. Sketch and label its image after a vertical compression by a factor of $\frac{1}{3}$, then a translation of 2 units up. State the domain of both functions.



$a = \frac{1}{3}$
$b = 1$
$h = 0$
$K = 2$

$y - 2 = \frac{1}{3} f(x)$

(x, y)	$\left(x, \frac{y}{3} + 2\right)$
(−5,3)	(−5,3)
(−2,−3)	(−2,1)
(1,0)	(1,2)
(4,6)	(4,4)

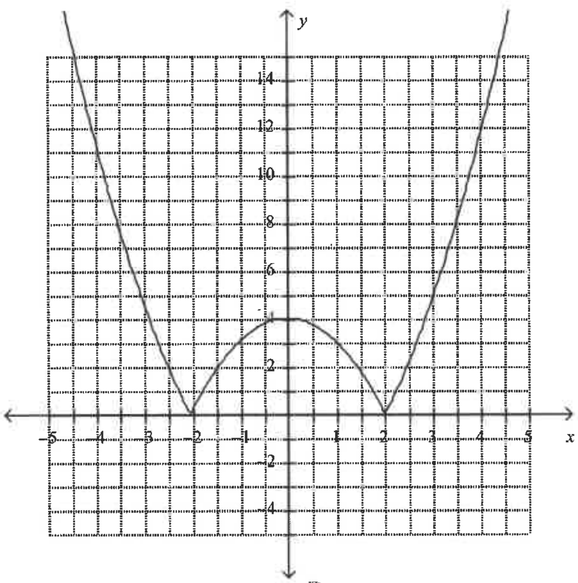
D: $\{x \mid -5 \le x \le 4, x \in \mathbb{R}\}$
R: $\{y \mid 1 \le y \le 4, y \in \mathbb{R}\}$

D: $\{x \mid -5 \le x \le 4, x \in \mathbb{R}\}$
R: $\{y \mid -3 \le y \le 6, y \in \mathbb{R}\}$

If you want another example of this go to example 1 on page 221.

Name: ______________________
Date: ______________________

Example 3: Here is the graph of y = f(x). Sketch the graph of $y - 6 = f(4(x + 2))$. State the domain and range of both functions.



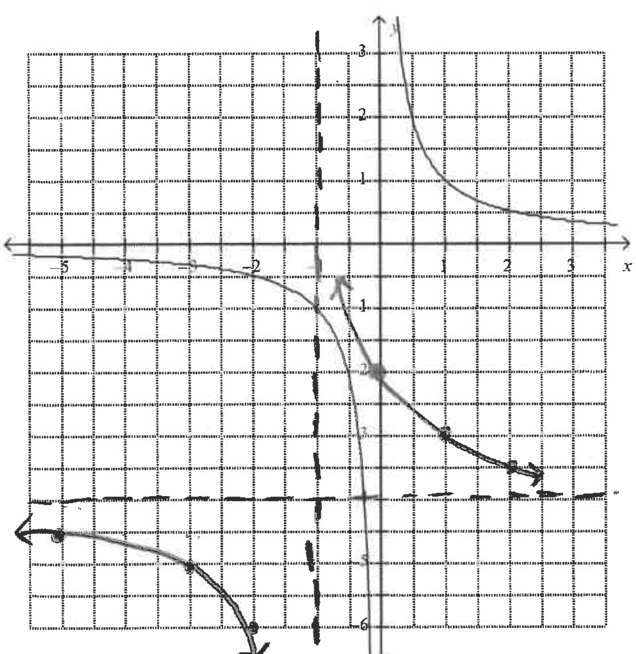
a = 1
b = 4
h = -2
K = 6

(x, y)	$\left(\frac{x}{4} - 2, y + 6\right)$
(-3.5, 8)	(-2.875, 14)
(-2, 0)	(-2.5, 6)
(0, 4)	(-2, 10)
(2, 0)	(-1.5, 6)
(3.5, 8)	(-1.125, 14)

D: {x | x ∈ ℝ}
R: {y | y ≥ 0, y ∈ ℝ}

D: {x | x ∈ ℝ}
R: {y | y ≥ 6, y ∈ ℝ}

Example 4: Here is the graph of y = f(x). Sketch the graph of $y + 4 = f(\frac{1}{2}(x + 1))$. State the domain and range of both functions.



a = 1
b = $\frac{1}{2}$
h = -1
K = -4

x, y	2x - 1, y - 4
(-2, -0.5)	(-5, -4.5)
(-1, -1)	(-3, -5)
(-0.5, -2)	(-2, -6)
(0.5, 2)	(0, -2)
(1, 1)	(1, -3)
(2, 0.5)	(3, -3.5)
x = 0	x = -1
y = 0	y = -4

D: {x | x ≠ 0, x ∈ ℝ}
R: {y | y ≠ 0, y ∈ ℝ}

D: {x | x ≠ -1, x ∈ ℝ}
R: {y | y ≠ -4, y ∈ ℝ}

Name: Page 229 # 8, 9, 10, 11, 12
Date: ______________________

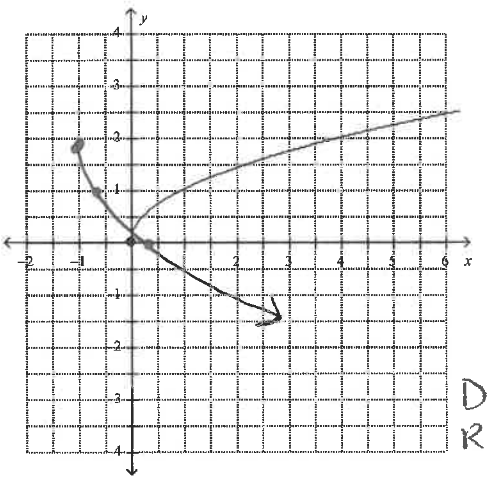
3.4 – Combining Transformations of Functions – Part 2

Sketching Transformations of Graphs

1. Choose lattice points
2. Determine all transformations, apply stretches and compressions, then reflections, then translations
3. Find your point $\left(\frac{x}{b} + h,\ ay + k\right)$
4. Fill in table of values of the image of each point, graph

Example 1: Use the graph of $y = \sqrt{x}$ to graph $y - 2 = -\sqrt{3x + 3}$. What are the domain and range of the transformed function?

$\underline{3x+3}$
3



$y - 2 = -\sqrt{3(x+1)}$

$a = -1$
$b = 3$
$h = -1$
$k = 2$

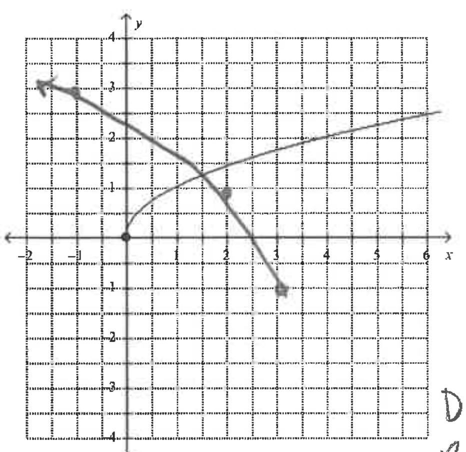
x, y	$\frac{x}{3} - 1,\ -y + 2$
$(0, 0)$	$(-1, 2)$
$(1, 1)$	$(-0.\overline{66}, 1)$
$(4, 2)$	$(0.\overline{33}, 0)$

D: $\{x \mid x \geq 0, x \in \mathbb{R}\}$
R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

D: $\{x \mid x \geq -1, x \in \mathbb{R}\}$
R: $\{y \mid y \leq 2, y \in \mathbb{R}\}$

Example 2: Use the graph of $y = \sqrt{x}$ to graph $y + 1 = 2\sqrt{-x + 3}$. What are the domain and range of the transformed function?

-1



$y + 1 = 2\sqrt{-1(x-3)}$

$a = 2$
$b = -1$
$h = 3$
$k = -1$

x, y	$-x + 3,\ 2y - 1$
$(0, 0)$	$(3, -1)$
$(1, 1)$	$(2, 1)$
$(4, 2)$	$(-1, 3)$

D: $\{x \mid x \geq 0, x \in \mathbb{R}\}$
R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

D: $\{x \mid x \leq 3, x \in \mathbb{R}\}$
R: $\{y \mid y \geq -1, y \in \mathbb{R}\}$

Example 3: The graph of $y = \sqrt{x}$ is vertically compressed by a factor of $\frac{1}{5}$, horizontally compressed by a factor of $\frac{1}{3}$, reflected in the y-axis, then translated 3 units right and 2 units down. Write an equation of the image graph in terms of *x*.

$a = 1/5$
$b = -3$
$h = 3$
$k = -2$

$y + 2 = \frac{1}{5}\sqrt{-3(x-3)}$

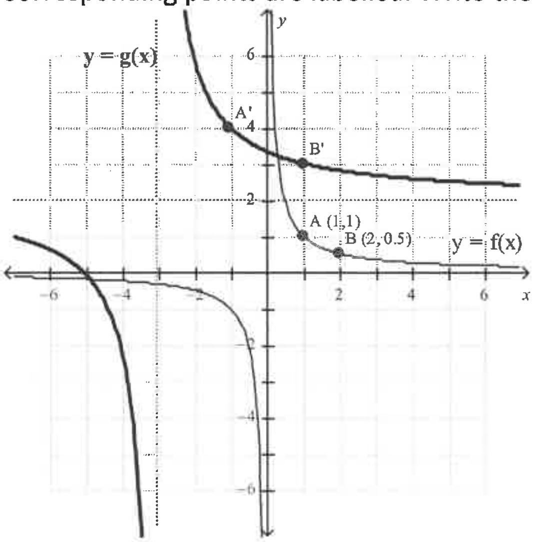
$y + 2 = \frac{1}{5}\sqrt{-3x + 9}$

Name: ______________________
Date: ______________________

Determining an Equation of a Function After a Transformation

1. Determine two corresponding points on each graph (A and B and A' and B')
2. Determine the horizontal and vertical distance between A and B
3. Determine the horizontal and vertical distance between A' and B'
4. Use this information to determine the a and b values using $\frac{x}{b} = x'$ and $ay = y'$
5. Use one point on both graph, apply stretches, compressions and reflections, and determine the translations using: $\frac{x}{b} + h = x'$ and $ay + k = y'$
6. Substitute all your values into $y - k = af(b(x - h))$

Example 4: The graph of $y = g(x)$ is the image of the graph $y = f(x)$ after a combination of transformations. Corresponding points are labelled. Write then verify an equation for the image graph in terms of the function f.



A(1,1) → A' (-1,4)
B(2,0.5) → B' (1,3)

$\left(\frac{x}{b} + h, \ ay + K\right)$

$(2x + h, 2y + K)$

Horizontal

A → B

$|1 - 2| = 1$

A' → B'

$|-1 - 1| = 2$

$\frac{x}{b} = x'$

$\frac{1}{b} = 2$

$b = 1/2$

Vertical

A → B

$|1 - 0.5| = 0.5$

A' → B'

$|4 - 3| = 1$

$ay = y'$

$\frac{a(0.5)}{0.5} = \frac{1}{0.5}$

$a = 2$

$x = 1$ $x' = -1$

$y = 1$ $y' = 4$

$2x + h = x'$

$2(1) + h = -1$

$-2 \quad -2$

$h = -3$

$2y + K = y'$

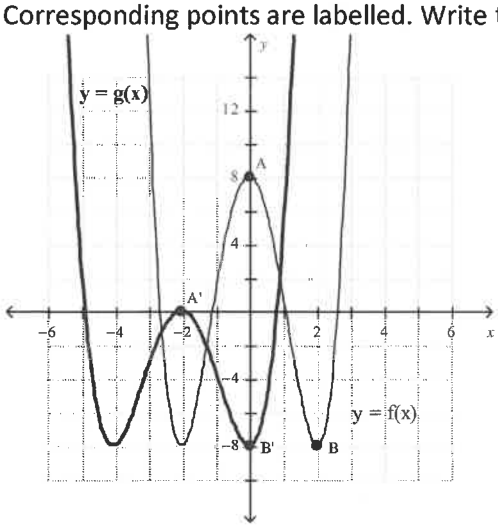
$2(1) + K = 4$

$-2 \quad -2$

$K = 2$

$y - 2 = 2f(0.5(x+3))$

Example 5: The graph of $y = g(x)$ is the image of the graph $y = f(x)$ after a combination of transformations. Corresponding points are labelled. Write then verify an equation for the image graph in terms of the function f.



A (0,8) → A' (-2,0)
B (2,-8) → B' (0,-8)

$\left(\frac{x}{b} + h, \ ay + K\right)$

$(x + h, 0.5y + K)$

Horizontal

A → B

$|0 - 2| = 2$

A' → B'

$|-2 - 0| = 2$

$\frac{x}{b} = x'$

$\frac{2}{b} = 2$

$b = 2/2 = 1$

Vertical

A → B

$|8 - (-8)| = 16$

A' → B'

$|0 - 8| = 8$

$ay = y'$

$\frac{a(16)}{16} = \frac{8}{16}$

$a = 0.5$

$x = 0$ $x' = -2$

$y = 8$ $y' = 0$

$x + h = x'$

$0 + h = -2$

$h = -2$

$0.5y + K = y'$

$0.5(8) + K = 0$

$4 + K = 0$

$-4 \quad -4$

$K = -4$

$y + 4 = 0.5(f(x+2))$

Name: Page 243 # 4, 5, 6, 9, 11, 12
Date:

3.5 – Inverse Relations – Part 1

Example 1: Solve for x in the following equations.

a) $y = 3x - 4$

$+4 \quad +4$

$\frac{y+4}{3} = \frac{\cancel{3}x}{\cancel{3}}$

$\frac{y+4}{3} = x$

b) $y = \frac{3x-5}{2}$ $\times 2 \quad \times 2$

$2y = 3x - 5$

$+5 \quad +5$

$\frac{2y+5}{3} = \frac{3x}{3}$

$\frac{2y+5}{3} = x$

c) $y = 3x^2 - 5$

$+5 \quad +5$

$\frac{y+5}{3} = \frac{3x^2}{3}$

$\pm\sqrt{\frac{y+5}{3}} = \sqrt{x^2}$

$\pm\sqrt{\frac{y+5}{3}} = x$

d) $y = 2(x-3)^2 + 4$

$-4 \quad -4$

$\frac{y-4}{2} = \frac{2(x-3)^2}{2}$

$\pm\sqrt{\frac{y-4}{2}} = \sqrt{(x-3)^2}$

$3 \pm\sqrt{\frac{y-4}{2}} = x - 3 \quad +3$

$3 \pm\sqrt{\frac{y-4}{2}} = x$

Inverse – Opposite or "reverse".

Example 2: Graph the function of $y = 2x + 4$ and $y = \frac{1}{2}x - 2$

$y = 2x + 4$
y-int = 4
slope = $\frac{2}{1}$

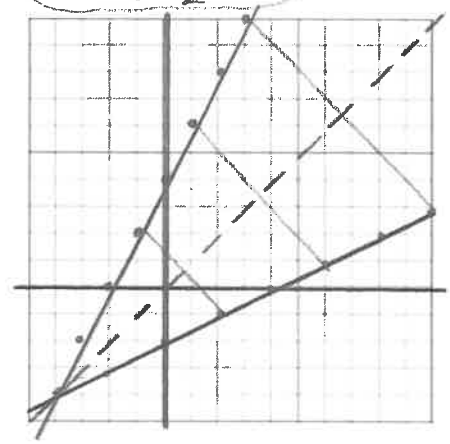
$y = \frac{1}{2}x - 2$
y-int = -2
slope = $1/2$

$f(x) = 2x+4$

x	y
-4	-4
-1	2
1	6
3	10

$f(y) = \frac{1}{2}x - 2$

x	y
-4	-4
2	-1
6	1
10	3



Reflecting in the Line $y = x$ – For a function $y = f(x)$, the graph of $x = f(y)$ is the image of the graph of $y = f(x)$ after a reflection in the line $y = x$. A point (x, y) on $y = f(x)$ corresponds to the point (y, x) on the graph of $x = f(y)$.

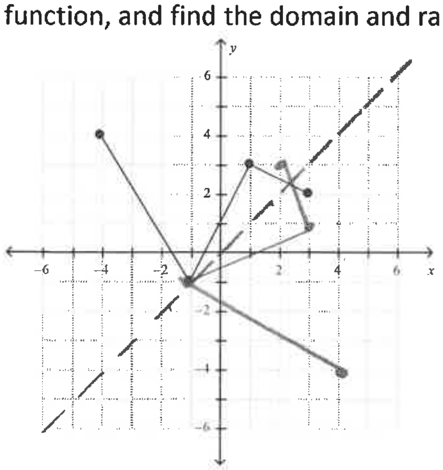
★ All inverse functions reflect on $y = x$

★ distance to $y = x$ is the same from f(x) and f(y) in order to be an inverse function

Sketching the Inverse of a Function Given a Graph

1. Sketch the line $y = x$
2. Choose points, (x, y), on the line for $y = f(x)$
3. Plot points for $x = f(y)$ as the points (y, x)

Example 3: Here is the graph of y = f(x). Sketch the graph of its inverse on the same graph, determine if it is a function, and find the domain and range of both functions.



f(x)

x	y
-4	4
-1	-1
1	3
3	2

invarient

f(y)

y	x
4	-4
-1	-1
3	1
2	3

D: $\{x \mid -1 \le x \le 4, x \in \mathbb{R}\}$
R: $\{y \mid -4 \le y \le 3, y \in \mathbb{R}\}$

D: $\{x \mid -4 \le x \le 3, x \in \mathbb{R}\}$
R: $\{y \mid -1 \le y \le 4, y \in \mathbb{R}\}$

If you want another example of this go to example 1 on page 237.

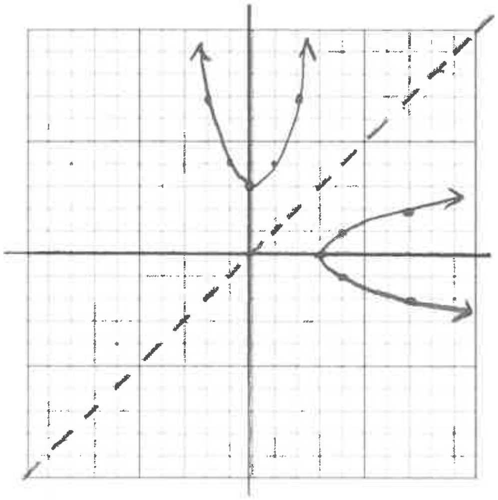
Name: ______________________
Date: ______________________

Domain and Range of a Function and its Inverse – The domain of y = f(x) is the range of x = f(y), and the range of y = f(x) is the domain of x = f(y).

Determining the Inverse Function

1. Interchange x and y in the equation
2. Solve for y

Example 4: Determine an equation of the inverse of $y = x^2 + 3$, sketch the graph as well as its inverse. Determine if the inverse is a function, and state the domain and the range.



$f(x) = x^2 + 3$

y-int = 3

x	y
1	1
2	4
3	9

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq 3, y \in \mathbb{R}\}$

$x = y^2 + 3$

$-3 \qquad -3$

$\sqrt{x-3} = \sqrt{y^2}$

$y = \pm\sqrt{x-3}$

f(x)

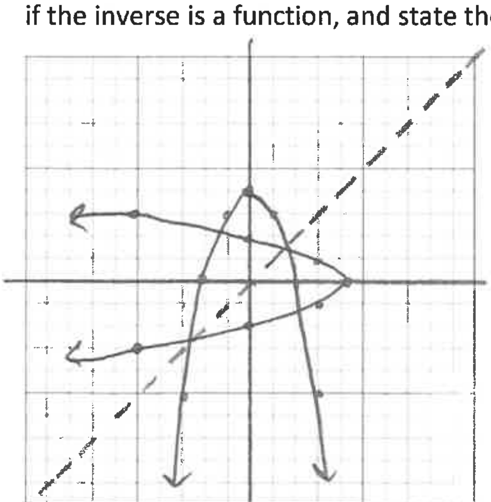
x	y
-2	7
-1	4
0	3
1	4
2	7

f(y)

x	y
7	-2
4	-1
3	0
4	1
7	2

x = f(y)
- is not a function
- doesn't pass VLT
- Restrict domain on y = f(x)

Example 5: Determine an equation of the inverse of $y = -x^2 + 4$, sketch the graph as well as its inverse. Determine if the inverse is a function, and state the domain and the range.



$f(x) = -x^2 + 4$

y-int = 4

x	y
1	-1
2	-4
3	-9

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq 4, y \in \mathbb{R}\}$

$x = -y^2 + 4$

$-4 \qquad -4$

$\frac{x-4}{-1} = \frac{-y^2}{-1}$

$\sqrt{-x+4} = \sqrt{y^2}$

$\pm\sqrt{-x+4} = y$

f(x)

x	y
-3	-5
-2	0
-1	3
0	4
1	3
2	0
3	-5

f(y)

y	x
-5	-3
0	-2
3	-1
4	0
3	1
0	2
-5	3

x = f(y)
- is not a function
- does not pass VLT
- Restrict domain on y = f(x)

Name: Page 244 #7, 8, 10, 13, 14
Date: ____________

3.5 – Inverse Relations – Part 2

Restrictions on Domain

1. Sketch the inverse of the function
2. Determine if the inverse is a function, if not, determine the point where it doesn't pass the Vertical Line Test
3. Restrict the domain and/or range to only portions of the function that pass the VLT

Example 1: Determine two ways to restrict the domain of $y = -x^2 + 5$ so that its inverse is a function. Write the equation of the inverse each time. Use a graph to illustrate each way.

f(x)

x	y
0	5
-2	1
2	1

f(y)

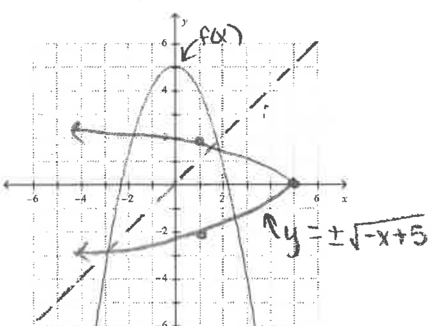
y	x
5	0
1	-2
1	2

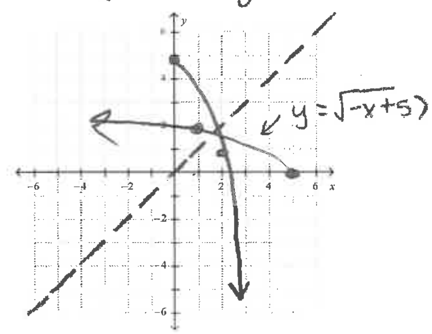
$x = -y^2 + 5$

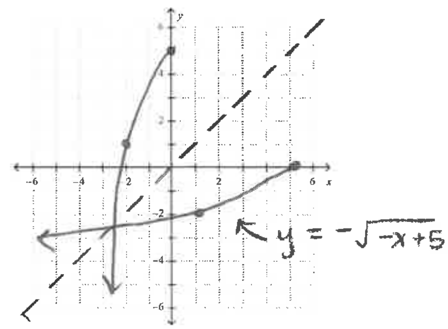
$x - 5 = -y^2$

$\sqrt{-x+5} = \sqrt{y^2}$

$\pm\sqrt{-x+5} = y$







D: $\{x \mid y = -x^2+5, x \in \mathbb{R}\}$

f(y) is not a function

D: $\{x \mid y = -x^2+5, x \geq 0, x \in \mathbb{R}\}$

D: $\{x \mid y = \sqrt{-x+5}, x \leq 5, x \in \mathbb{R}\}$

D: $\{x \mid y = -x^2+5, x \leq 0, x \in \mathbb{R}\}$

D: $\{x \mid y = -\sqrt{-x+5}, x \leq 5, x \in \mathbb{R}\}$

Example 2: Determine two ways to restrict the domain of $y = (x-1)^2 + 3$ so that its inverse is a function. Write the equation of the inverse each time. Use a graph to illustrate each way.

f(x)

x	y
-1	7
1	3
3	7

f(y)

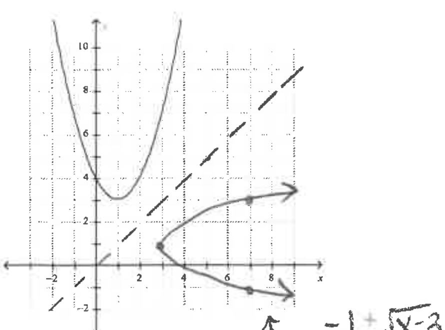
y	x
7	-1
3	1
7	3

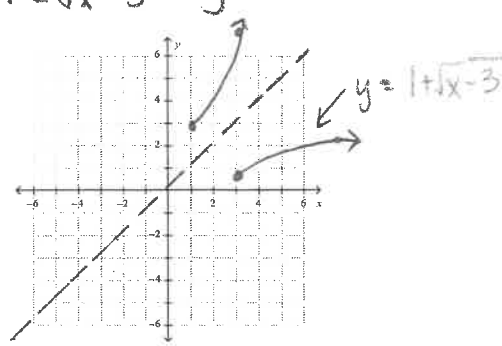
$x = (y-1)^2 + 3$

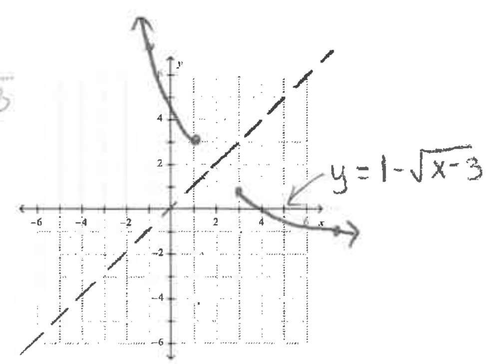
$\sqrt{x-3} = \sqrt{(y-1)^2}$

$1 \pm \sqrt{x-3} = y - 1$

$1 \pm \sqrt{x-3} = y$







D: $\{x \mid y = (x-1)^2+3, x \in \mathbb{R}\}$

f(y) is not a function

D: $\{x \mid y = (x-1)^2+3, x \geq 1, x \in \mathbb{R}\}$

D: $\{x \mid y = 1+\sqrt{x-3}, x \geq 3, x \in \mathbb{R}\}$

D: $\{x \mid y = (x-1)^2+3, x \leq 1, x \in \mathbb{R}\}$

D: $\{x \mid y = 1-\sqrt{x-3}, x \geq 3, x \in \mathbb{R}\}$

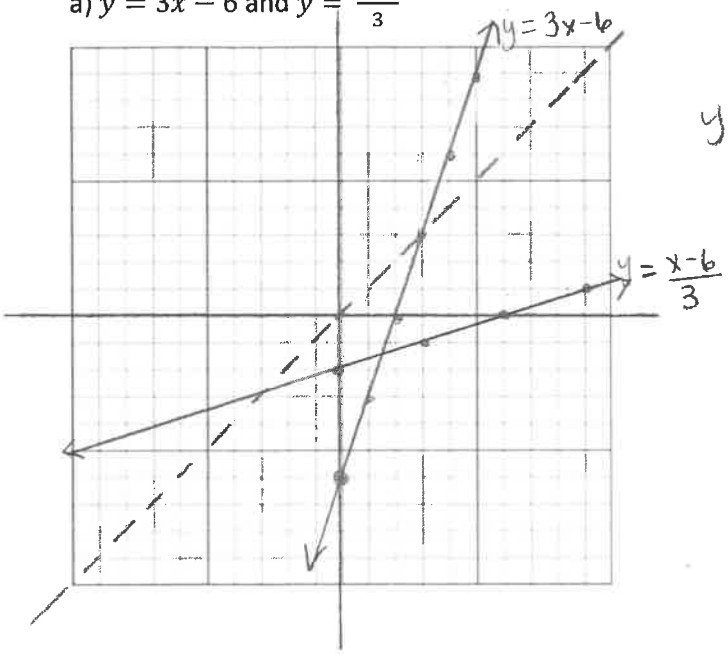
Name: ____________________
Date: ____________________

Determining Whether Functions are Inverses

1. Interchange x and y in one equation
2. Solve for y
3. If the equations match, then they are inverses of each other.

Example 3: Determine whether the functions in each pair are inverses of each other algebraically and graphically

a) $y = 3x - 6$ and $y = \frac{x-6}{3}$



$y = \frac{x-6}{3}$

$y = \frac{x}{3} - 2$

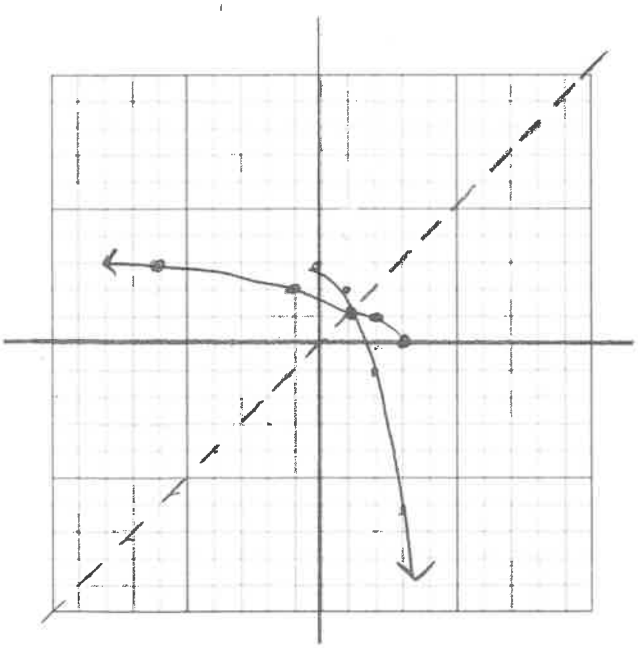
$x \overset{+6}{=} 3y \overset{+6}{-6}$

$\frac{x+6}{3} = \frac{3y}{3}$

$\frac{x+6}{3} = y$

Not the inverse

b) $y = -x^2 + 3, x \geq 0$ and $y = \sqrt{3 - x}$



$y = -x^2 + 3$

v $(0, 3)$

x	y
1	-1
2	-4
3	-9

$y = \sqrt{\frac{3-x}{-1}}$

$y = \sqrt{-1(x+3)}$

x	y
9	3
4	2
1	1
0	0

* Reflected on x-axis
* Translated 3 right

$x \overset{-3}{=} -y^2 + 3$
$ -3$

$\frac{x-3}{-1} = \frac{-y^2}{-1}$

$\sqrt{-x+3} = \sqrt{y^2}$

$\pm\sqrt{-x+3} = y$

or

$\pm\sqrt{3-x} = y$

It is the inverse

If you want another example of this go to example 4 on page 241.