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Unit 5 – Exponential and Logarithmic Functions – Practice Test

1. The pH of a solution can be described by the equation $\text{pH} = -\log\left[\text{H}^+\right]$, where $\left[\text{H}^+\right]$ is the hydrogen-ion concentration in moles/litre.

a) The hydrogen-ion concentration in a sample of watermelon is 2.0×10^{-6} moles/litre. Determine the pH of the watermelon, to the nearest tenth.

$pH = -\log(2(10)^{-6})$

$pH = 5.6989....$

$pH = 5.7$

b) A sample of lemon juice has a pH of 2.1. Determine the hydrogen-ion concentration of the lemon juice, to four decimal places.

$\frac{2.1}{-1} = \frac{-\log(H^+)}{-1}$

$-2.1 = \log(H^+)$

$10^{-2.1} = H^+$

$0.0079 = H^+$

2. The decibel scale measures the intensity of sound. The loudness of a sound, L decibels (dB), can be determined using the function $L = 10\log\left(\frac{I}{I_0}\right)$, where I is the intensity of the sound and I_0 is the intensity of the quietest sound that can be detected. Determine the loudness of a sound, in decibels, that is one-fourth as intense as a sound with loudness 60 dB. Give the answer to the nearest whole number.

$\frac{60}{10} = \frac{10}{10}\log\left(\frac{I}{I_0}\right)$

$6 = \log\left(\frac{I}{I_0}\right)$

$10^6 = \frac{I}{I_0}$

$10^6 I_0 = I$

$I = \frac{1}{4}(10^6 I_0)$

$L = 10\log\left(\frac{\frac{1}{4}10^6 I_0}{I_0}\right)$

$= 10\log\left(\frac{1}{4}10^6\right)$

$= 53.9794...$ $= 54$ dB

3. Suppose a student graduates with a student loan of \$17 000. The loan payments are \$220 per month at 2.5% annual interest, compounded monthly. To the nearest month, how long will it take the student to repay the loan? How much interest will they pay?

$A = 17\,000$
$P = 220$
$i = 0.025$
$n = 12$
$t =$
$1 + \frac{i}{n} = 1 + \frac{0.025}{12}$

$A = P\left(\frac{1-(1+\frac{i}{n})^{-nt}}{\frac{i}{n}}\right)$

$\frac{A}{P}\cdot\left(\frac{i}{n}\right) = 1-(1+\frac{i}{n})^{-nt}$

$-\left(\frac{A}{P}\cdot\left(\frac{i}{n}\right)-1\right) = +(1+\frac{i}{n})^{-nt}$

$\frac{\log\left(-\frac{A}{P}\cdot\frac{i}{n}+1\right)}{-n\log(1+\frac{i}{n})} = \frac{-nt\log(1+\frac{i}{n})}{-n\log(1+\frac{i}{n})}$

$t = \frac{\log\left(-\frac{(17000)}{220}\cdot\left(\frac{0.025}{12}\right)+1\right)}{-12\log\left(1+\frac{0.025}{12}\right)}$

$t = 7.0283...$ years (12)

$= 84.340...$ monthly payments (220)

\$18554.90 total paid

18554.90
−17000
1 554.90 interest

★ I used the manipulated formula I found from Question 3 to shorten my work.

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4. Two students each graduate with a student loan of $15 000 at 2.8% annual interest. Both students make payments totalling $3588 per year. Student A makes payments of $299 per month, and the interest is compounded every month. Student B makes payments of $138 every two weeks, and the interest is compounded every two weeks. Compare the lengths of times it takes each student to repay the loan. Is one payment plan significantly better than the other? Explain.

Student A

A = 15000
P = 299
i = 0.028
n = 12
t = ?

$$t = \frac{\log\left(-\frac{A}{P}\cdot\frac{i}{n}+1\right)}{-n\log\left(1+\frac{i}{n}\right)}$$

$$t = \frac{\log\left(-\frac{15000}{299}\cdot\frac{0.028}{12}+1\right)}{-12\log\left(1+\frac{0.028}{12}\right)}$$

t = 4.451... years (12)

= 53.41... monthly payments (299)

= 15 971.68 = 971.68 interest

Student B

A = 15000
P = 138
i = 0.028
n = 26

$$t = \frac{\log\left(-\frac{15000}{138}\cdot\left(\frac{0.028}{26}\right)+1\right)}{-26\log\left(1+\frac{0.028}{26}\right)}$$

t = 4.4486... years (26)

115.66... weekly payments (138)

$15 961.67 = $961.67 interest

student B pays slightly less interest.

5. A principal of $400 is invested at 2% annual interest, compounded quarterly. To the nearest year, when will the amount be $600?

A = 600
P = 400
i = 0.02
n = 4

$$A = P\left(1+\frac{i}{n}\right)^{nt}$$

$$\frac{600}{400} = \frac{400\left(1+\frac{0.02}{4}\right)^{4t}}{400}$$

$$\log 1.5 = \log\left(1+\frac{0.02}{4}\right)^{4t}$$

$$\frac{\log 1.5}{4\log\left(1+\frac{0.02}{4}\right)} = \frac{4t\log\left(1+\frac{0.02}{4}\right)}{4\log\left(1+\frac{0.02}{4}\right)}$$

20.323... = t

6. Solve: $\frac{125}{5} = \frac{5\left(3^{x+4}\right)}{5}$ Give the solution to the nearest hundredth.

$$\log 25 = \log 3^{x+4}$$

$$\frac{\log 25}{\log 3} = \frac{(x+4)\log 3}{\log 3}$$

$$\frac{\log 25}{\log 3} - 4 = x$$

-1.07005... = x

-1.07 = x

7. Solve: $5^x = 7^{x-2}$ Give the solution to the nearest hundredth.

$$\log 5^x = \log 7^{x-2}$$

$$x\log 5 = (x-2)\log 7$$

$$x\log 5 = x\log 7 - 2\log 7$$

$$-x\log 7 \quad -x\log 7$$

$$x\log 5 - x\log 7 = -2\log 7$$

$$x(\log 5 - \log 7) = -2\log 7$$

$$\frac{x\log\left(\frac{5}{7}\right)}{\log\left(\frac{5}{7}\right)} = \frac{-2\log 7}{\log 5/7}$$

$$x = \frac{-2\log 7}{\log(5/7)}$$

x = 11.5665...

x = 11.57

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8. Solve: $9^{x+1} = 8^{x+2}$ Give the solution to the nearest hundredth.

$\log 9^{x+1} = \log 8^{x+2}$
$(x+1)\log 9 = (x+2)\log 8$
$x\log 9 + \log 9 = x\log 8 + 2\log 8$
$x\log 9 - x\log 8 = 2\log 8 - \log 9$

$x(\log 9 - \log 8) = \log 8^2 - \log 9$
$\frac{x\log(9/8)}{\log(9/8)} = \frac{\log(8^2/9)}{\log(9/8)}$
$x = \frac{\log(64/9)}{\log(9/8)} = 16.6548\ldots = 16.65$

9. Solve: $\log_2(x+11) - \log_2(x+2) = 1 + \log_2(x-3) - \log_2(x-2)$

$\log_2 \frac{(x+11)}{(x+2)} = \log_2 2^1 + \log_2(x-3) - \log_2(x-2)$

$\log_2 \frac{(x+11)}{(x+2)} = \log_2 \frac{2(x-3)}{x-2}$ * when they are equal with the same base you can write without log. * this skips the steps

$(x-2)(x+2) \cdot \frac{(x+11)}{(x+2)} = \frac{(2x-6)(x+2)}{(x-2)} \cdot (x-2)$

$(x-2)(x+11) = (2x-6)(x+2)$
$x^2 + 9x - 22 = 2x^2 - 2x - 12$
$-x^2 - 9x + 22 \quad -x^2 - 9x + 22$
$0 = x^2 - 11x + 10$ $\quad -10 + -1 = -11$, $-10 \cdot -1 = 10$
$0 = (x-10)(x-1)$
$x = 10$ ~~$x = 1$~~ ← extraneous because $\log_2(1-3) = \log_2(-2)$ X, $\log_2(1-2) = \log_2(-1)$ X

10. Solve: $\log(2x-20) + \log(x-3) = \log(x-6) + \log(x-6)$

$\log(2x-20)(x-3) = \log(x-6)(x-6)$
$\log(2x^2 - 26x + 60) = \log(x^2 - 12x + 36)$
$2x^2 - 26x + 60 = x^2 - 12x + 36$
$-x^2 + 12x - 36 \quad -x^2 + 12x - 36$
$-12 + -2 = -14$, $-12 \cdot -2 = 24$ $\quad x^2 - 14x + 24 = 0$

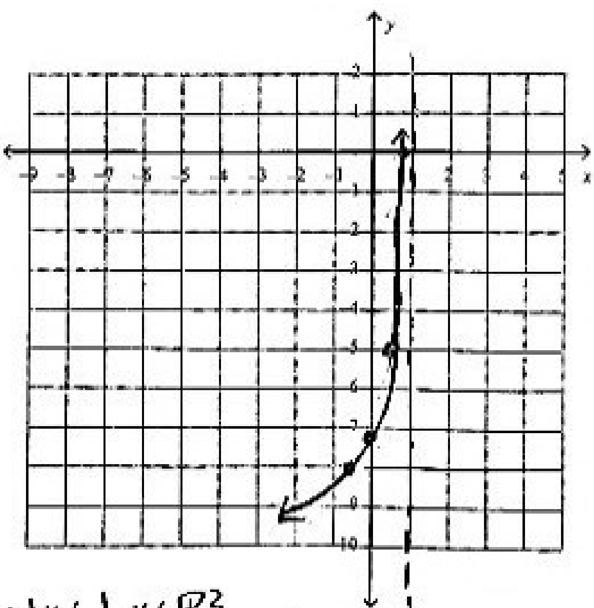
$(x-12)(x-2) = 0$
$x = 12$ ~~$x = 2$~~ $= 0$
$x = 12$

$\log(2(2)-20) = \log(-16)$
$\log(2-3) = \log(-1)$
$\log(2-6) = \log(-4)$
*all extraneous

11. Determine the equation of the <u>vertical asymptote</u> of the graph of $y = -4\log_4(x+2)$.

Original asymptote $x = 0$
translation 2 units left $\quad \therefore$ asymptote: $x = -2$

12. a) Graph $y = -3\log_6(-4(x-1)-5)$ on the grid below.
b) Identify the intercepts, the equations of any asymptotes, and the domain and range of the function.



$y = \log_6 x$

x	y
1	0
6	1
x=0	

$y = -3\log_6(-4(x-1)-5)$

$\frac{x}{-4}+1$	$-3y-5$
$-\frac{1}{4}+1 = \frac{3}{4}$	-5
$-\frac{6}{4}+1 = -\frac{1}{2}$	-8
$x = 1$	

$d = -4 \quad c = -3$
$h = 1 \quad k = -5$

$0 = -3\log_6(-4(x-1)) - 5$ (+5, +5)
$\frac{5}{-3} = \log_6(-4(x-1))$
$\frac{6^{-5/3}}{-4} + 1 = \frac{-4(x-1)}{-4} + 1$
$\frac{6^{-5/3}}{-4} + 1 = x$
$0.987\ldots = x$

$y = -3\log_6(-4(0-1)) - 5$
$y = -3\log_6(4) - 5$
$y = -3\frac{\log 4}{\log 6} - 5$
$y = -7.32\ldots$

D: $\{x \mid x < 1, x \in \mathbb{R}\}$
R: $\{y \mid y \in \mathbb{R}\}$
asymptote: $x = 1$
x-int: $x = 0.987\ldots$ $\quad$ $y = -7.32\ldots$

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13, Write as a single logarithm: $\frac{8}{3}\log x + 6\log y$

$\log x^{8/3} + \log y^{6}$

$\log x^{8/3} y^{6}$

14. Write this expression in terms of $\log a$, $\log b$, and $\log c$. $\log\left(ab^{\frac{7}{4}}c^{5}\right)$

$\log a \cdot \log b^{7/4} \cdot \log c^{5}$

$\log a \cdot 7/4 \log b \cdot 5\log C$

15. Write as a single logarithm: $2\log(x+5) + 2\log(x-7) - \log(x^2 - 2x - 35)$

$\log(x+5)^2 + \log(x-7)^2 - \log(x^2-2x-35)$

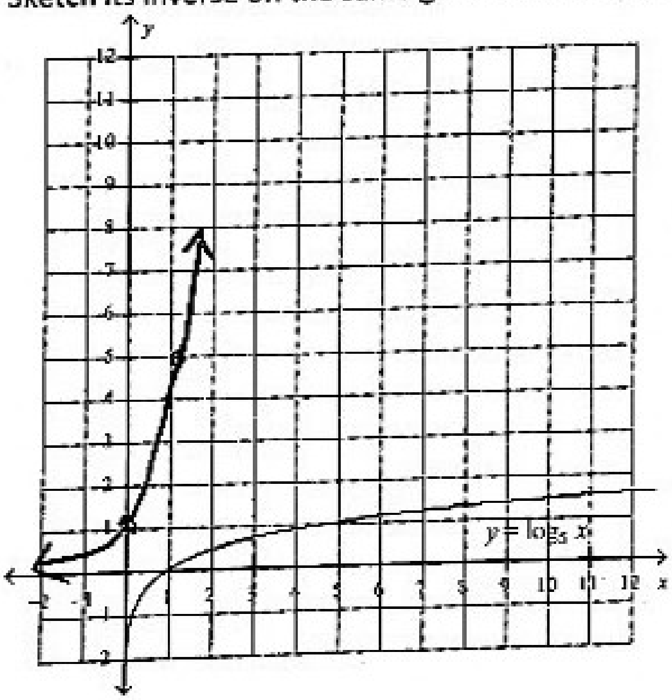
$\log\left(\frac{(x+5)^2(x-7)^2}{(x-7)(x+5)}\right)$

$\log\left(\frac{(x+5)(x+5)(x-7)(x-7)}{(x+5)(x-7)}\right) \rightarrow \log(x+5)(x-7)$

or

$\log(x^2-2x-35)$

16. The graph of $y = \log_5 x$ is shown below.
Sketch its inverse on the same grid. Label the graph with its equation.



$y = \log_5 x$

x	y
1	0
5	1
x=0	

$y = 5^x$

x	y
0	1
1	5
.	y=0

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17. Evaluate: $\log_7\left(49\sqrt[4]{343}\right)$

$\log_7\left(7^2 \cdot \sqrt[4]{7^3}\right)$

$\log_7\left(7^2 \cdot 7^{3/4}\right)$

$\log_7\left(7^{2+3/4}\right)$

$\log_7 7^{11/4}$

$\rightarrow 7^x = 7^{11/4}$

$x = 11/4$

18. To the nearest hundredth, estimate the value of $\log_3 5.1$. Show your work, do not evaluate with a calculator.

$3^2 = 9$

$3^1 = 3$

$3^{1.5} = 5.196$

$3^{1.4} = 4.65$

$3^{1.45} = 4.918$

$3^{1.47} = 5.027$

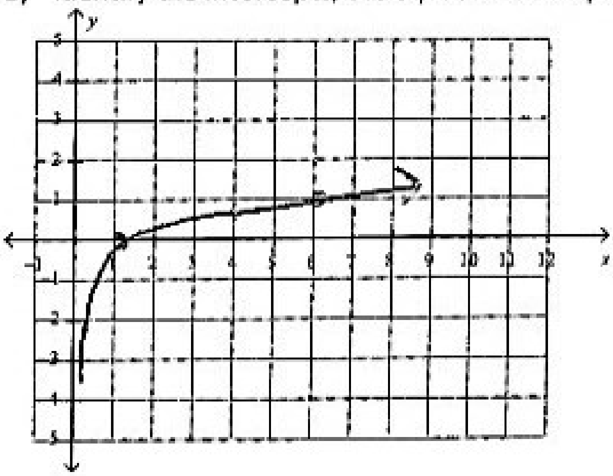
$3^{1.46} = 4.97$

$\rightarrow \log_3 5.1 = 1.46$

19. Write 7 as a logarithm with base 2.

20. a) Graph $y = \log_6 x$.

b) Identify the intercepts, the equations of any asymptotes, and the domain and range of the function.



x	y
1	0
6	1
x = 0	

x-int = 1
y-int = none
asymptote: x = 0
$D: \{x \mid x > 0, x \in \mathbb{R}\}$
$R: \{y \mid y \in \mathbb{R}\}$

21. Solve: $\left(\frac{1}{16}\right)^{x+4} = \left(\sqrt[3]{256}\right)^x$

$\left(\frac{1}{4^2}\right)^{x+4} = \left(\sqrt[3]{4^4}\right)^x$

$4^{-2(x+4)} = 4^{4/3(x)}$

$-2(x+4) = \frac{4}{3}x$

$\rightarrow 3\left(-2x+4\right) = \left(\frac{4}{3}x\right)3$

$-6x + 12 = 4x$
$+6x \quad +6x$

$\frac{12}{10} = \frac{10x}{10}$

$12/10 = x$

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22. Solve: $\left(\frac{1}{9}\right)^x = 3\sqrt[3]{81}$

$\left(\frac{1}{3^2}\right)^x = 3\sqrt[3]{3^4}$

$3^{-2(x)} = 3^1 \cdot 3^{4/3}$

$3^{-2x} = 3^{1+4/3}$

$3^{-2x} = 3^{7/3}$

$\frac{-2x}{-2} = \frac{7/3}{-2}$

$x = \frac{-7}{6}$

23. Solve: $\left(\sqrt[3]{5}\right)^{x-5} = \sqrt[4]{125}$

$\left(5^{1/3}\right)^{x-5} = \sqrt[4]{5^3}$

$5^{1/3(x-5)} = 5^{3/4}$

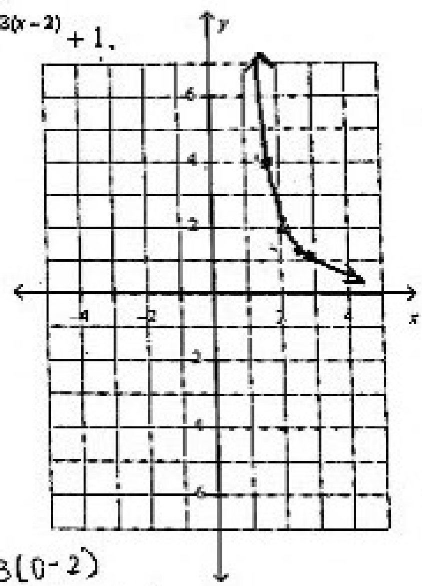
$12\left(\frac{1}{3}(x-5)\right) = \left(\frac{3}{4}\right)12$

$4x - 60 = 9$
$+60 \quad +60$

$\frac{4x}{4} = \frac{69}{4}$

$x = \frac{69}{4}$

24. a) Use transformations to sketch the graph of the exponential function $y = 3^{-3(x-2)} + 1$.
b) Determine:
i) whether the function is increasing or decreasing
ii) the intercepts
iii) the equation of the asymptote
iv) the domain of the function
v) the range of the function



$y = 3^x$

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

$y = 3^{-3(x-2)} + 1$

$-\frac{x}{3} + 2$	$y + 1$
$\frac{2}{3} + 2 = \frac{8}{3}$	$\frac{1}{9} + 1 = 10/9$
$\frac{1}{3} + 2 = \frac{7}{3}$	$1/3 + 1 = 4/3$
2	2
$-\frac{1}{3} + 2 = \frac{5}{3}$	4
$-\frac{2}{3} + 2 = 4/3$	10

$d = -3 \quad c = 1$
$h = 2 \quad k = 1$

$y = 3^{-3(0-2)} + 1$
$y = 730$

x-int = none   asymptote: $y = 0$
y-int = 730
D: $\{x \mid x \in \mathbb{R}\}$ R: $\{y \mid y > 0, y \in \mathbb{R}\}$

25. What are you going to work on better next unit?

DO QUESTIONS FROM THE TEXT BOOK