

Unit 3 – Transforming Graphs of Functions – Practice Test

1. The function $y = f(x)$ has domain $-9 \leq x \leq 5$ and range $-7 \leq y \leq 11$. What are the domain and range of $y + 4 = f(x + 2)$?

Vertical translation -4 (down)
Horizontal translation -2 (left)

$$(x - 2, y - 4)$$

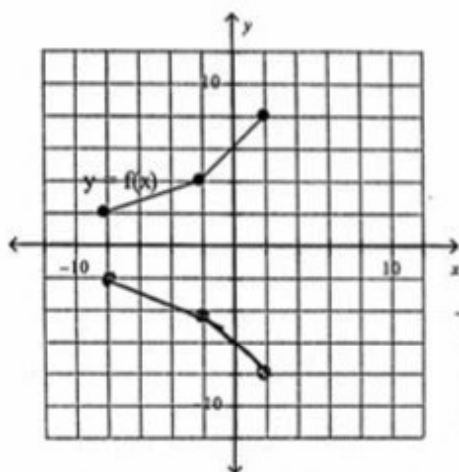
$$D: \{x \mid -9 - 2 \leq x \leq 5 - 2, x \in \mathbb{R}\}$$

$$R: \{y \mid -7 - 4 \leq y \leq 11 - 4, y \in \mathbb{R}\}$$

$$D: \{x \mid -11 \leq x \leq 3, x \in \mathbb{R}\}$$

$$R: \{y \mid -11 \leq y \leq 7, y \in \mathbb{R}\}$$

2. Here is the graph of $y = f(x)$. On the same grid, sketch the graph of $y = -f(x)$.



$f(x)$	
x	y
-8	2
-2	4
2	8

$-f(x)$	
x	-y
-8	-2
-2	-4
2	-8

* Reflection in the x-axis

$$D: \{x \mid -8 \leq x \leq 8, x \in \mathbb{R}\}$$

$$D: \{x \mid -8 \leq x \leq 8, x \in \mathbb{R}\}$$

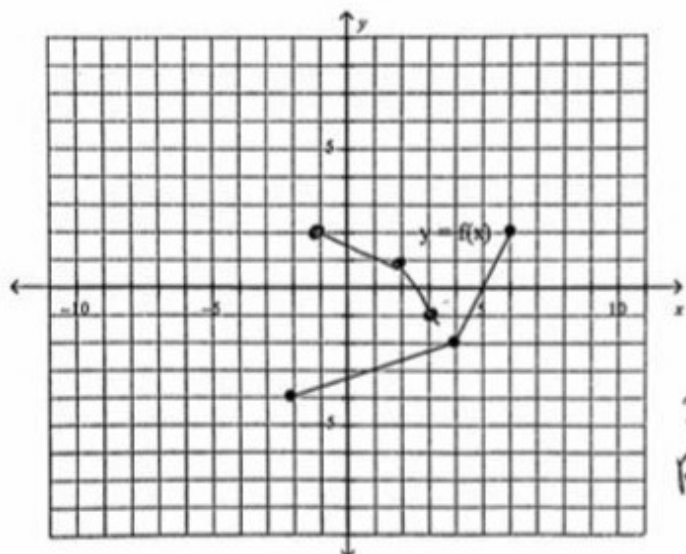
$$R: \{y \mid 2 \leq y \leq 8, y \in \mathbb{R}\}$$

$$R: \{y \mid -8 \leq y \leq -2, y \in \mathbb{R}\}$$

3. Here is the graph of $y = f(x)$. On the same grid, sketch the graph of $y = -\frac{1}{2}f(2x)$. State the domain and range of each function.

$$a = -\frac{1}{2}$$

$$b = 2$$



$f(x)$	
x	y
-2	-4
4	-2
6	2

$y = -\frac{1}{2}f(2x)$	
$\frac{1}{2}x$	$-\frac{1}{2}y$
-1	2
2	1
3	-1

$$D: \{x \mid -2 \leq x \leq 6, x \in \mathbb{R}\}$$

$$R: \{y \mid -4 \leq y \leq 2, y \in \mathbb{R}\}$$

$$D: \{x \mid -1 \leq x \leq 3, x \in \mathbb{R}\}$$

$$R: \{y \mid -1 \leq y \leq 2, y \in \mathbb{R}\}$$

4. Determine the equation of the function $y = \frac{(x-2)^3}{x-4}$ after a vertical compression by a factor of $\frac{1}{2}$, a horizontal compression by a factor of $\frac{1}{2}$, a reflection in the y-axis, and a reflection in the x-axis.

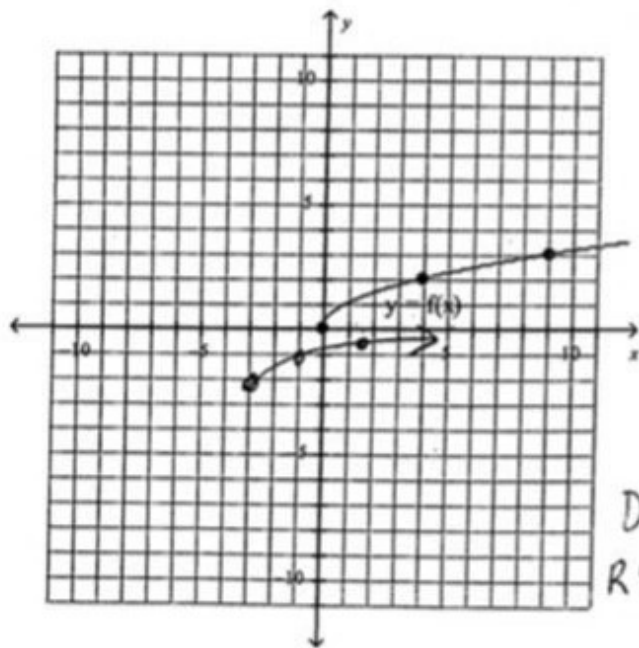
$a = \frac{1}{2} = 0.5$
 $b = -2$
 $h = 0$
 $k = 0$

$$y = -0.5 \left(\frac{(-2x-2)^3}{(-2x-4)} \right)$$

5. Describe how the graph of $y - 3 = \frac{1}{2} f(-2(x-3))$ is related to the graph of $y = f(x)$.

- Vertical Translation +3 (up)
- Horizontal Translation +3 (right)
- Vertical Compression by a factor of $\frac{1}{2}$
- horizontal compression by a factor of $\frac{1}{2}$
- reflected on y-axis

6. Here is the graph of $y = f(x)$. The graph of $y = f(x)$ is transformed by: a vertical compression by a factor of $\frac{1}{2}$, a horizontal compression by a factor of $\frac{1}{2}$, no reflection, and a translation of 2 units left and 2 units down. Write an equation of the image graph in terms of the function f . Sketch the image graph, then state its domain and range.



$a = \frac{1}{2} = 0.5$
 $b = 2$
 $h = -3$
 $k = -2$

$f(x)$	
x	y
0	0
4	2
9	3

$D: \{x \mid x \geq 0, x \in \mathbb{R}\}$
 $R: \{y \mid y \geq 0, y \in \mathbb{R}\}$

$$y - k = a f(b(x - h))$$

$$y + 2 = 0.5 f(2(x + 3))$$

$\frac{x}{b} + h$	$ay + k$
$\frac{x}{2} - 3$	$0.5y - 2$
-3	-2
-1	-1
1.5	-0.5

$D: \{x \mid x \geq -3, x \in \mathbb{R}\}$
 $R: \{y \mid y \geq -2, y \in \mathbb{R}\}$

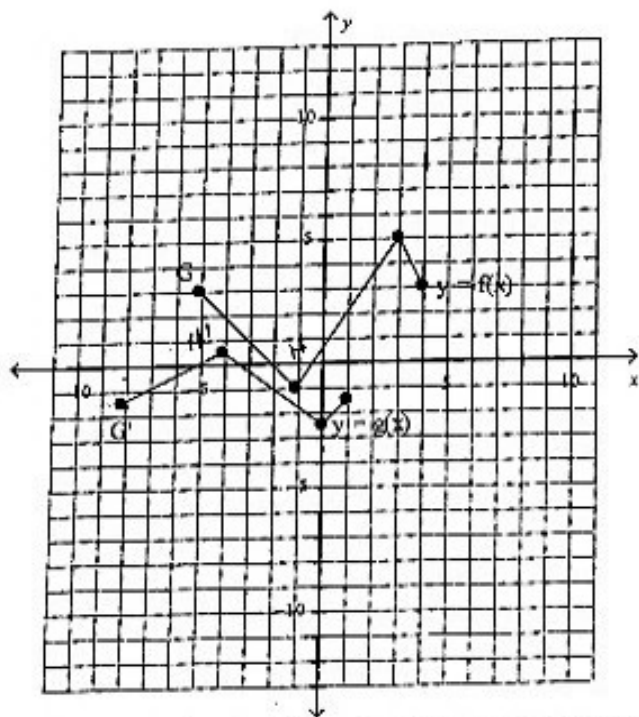
7. The graph of $y = |x|$ is vertically compressed by a factor of $\frac{1}{3}$, horizontally compressed by a factor of $\frac{1}{3}$, reflected in the y-axis, then translated 3 units left and 4 units down. Write an equation of the image graph in terms of x .

$a = \frac{1}{3}$
 $b = -3$
 $h = -3$
 $k = -4$

$$y - k = a f(b(x - h))$$

$$y + 4 = \frac{1}{3} \left| -3(x + 3) \right|$$

8. The graph of $y = g(x)$ is the image of the graph of $y = f(x)$ after a combination of transformations.
Write an equation for the transformations.



$G(-5, 3) \rightarrow H(-1, -1)$
 $G'(-8, -1.5) \rightarrow H'(4, 0.5)$

Horizontal!
 $G \rightarrow H$
 $|-5 - (-1)| = 4$
 $G' \rightarrow H'$
 $|-8 - (-4)| = 4$

$\frac{x}{b} = x'$
 $\frac{4}{b} = \frac{4}{4}$
 $b = \frac{4}{4} = 1$

Vertical!
 $G \rightarrow H$
 $|3 - (-1)| = 4$
 $G' \rightarrow H'$
 $|-1.5 - 0.5| = 1$

$ay = y'$
 $a(4) = 1$
 -4
 $a = -1/4$
 *reflection on the axis

*reflection on x-axis
 → make a neg
 $x = -5 \quad x' = -1$
 $y = 3 \quad y' = -1$

$x + h = x'$
 $-5 + h = -1$
 $+5$
 $h = -3$

$-\frac{1}{4}y + k = y'$
 $-\frac{1}{4}(3) + k = -1.5$
 $-0.75 + k = -1.5$
 $+0.75$
 $k = -0.75$

$y + 0.75 = -0.25(x + 3)$

9. Determine whether these functions are inverses of each other.

$y = \frac{7x+6}{2}$

$y = \frac{2x+6}{7}$

$x^2 = \frac{7y+6}{2} x^2$

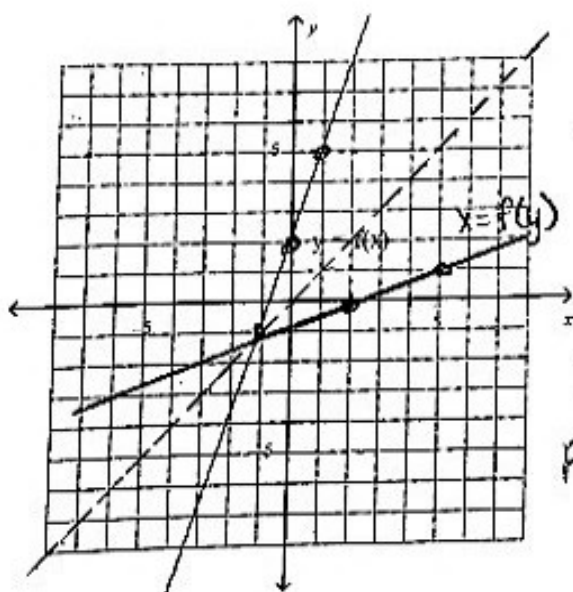
$\frac{2x-6}{7} = \frac{7y}{7}$

$2x = 7y + 6$

$\frac{2x-6}{7} = y$

These are not inverses of each other.

10. Here is the graph of $y = f(x)$. On the same grid, sketch the graph of its inverse.



$f(x)$

x	y
-1	-1
0	2
1	5

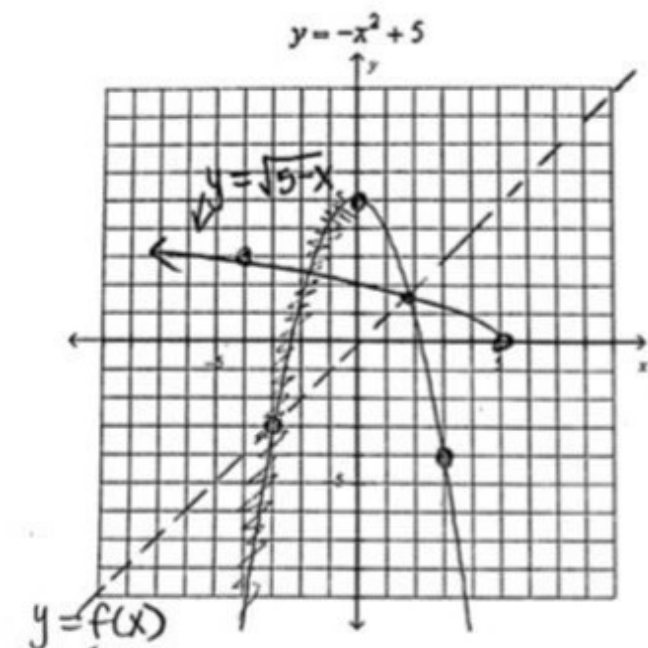
$D: \{x | x \in \mathbb{R}\}$
 $R: \{y | y \in \mathbb{R}\}$

$f(y)$

x=y	y=x
-1	-1
2	0
5	1

$D: \{y | x \in \mathbb{R}\}$
 $R: \{y | y \in \mathbb{R}\}$

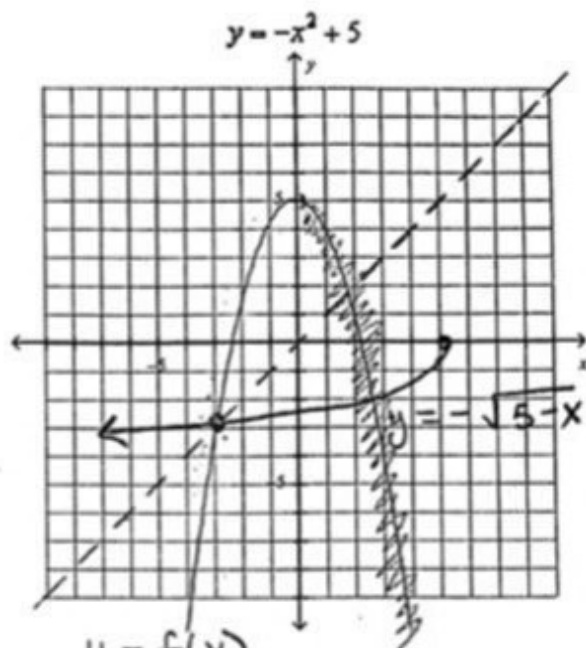
12. Determine two ways to restrict the domain of $y = -x^2 + 5$ so that its inverse is a function. Write the equation of the inverse each time. Use a graph to illustrate each way. State the domain of the restricted $f(x)$, and state the domain of the inverse function.



$y = f(x)$
 $D: \{x \mid y = -x^2 + 5, x \geq 0, x \in \mathbb{R}\}$

$x = f(y)$

$D: \{x \mid y = \sqrt{5-x}, x \leq 5, x \in \mathbb{R}\}$



$y = f(x)$

$D: \{x \mid y = -x^2 + 5, x \leq 0, x \in \mathbb{R}\}$

$x = f(y)$

$D: \{x \mid y = -\sqrt{5-x}, x \leq 5, x \in \mathbb{R}\}$

Equation

$$x = -y^2 + 5$$

$$\frac{x-5}{-1} = \frac{-y^2}{-1}$$

$$-x + 5 = y^2$$

$$\sqrt{y^2} = \sqrt{5-x}$$

$$y = \pm \sqrt{5-x}$$

↑
not a function
with \pm for
 $x = f(y)$

$f(x)$

x	y
-3	-3
0	5
3	-4

$x \geq 0$

$f(y)$

x=y	y=x
-3	-3
5	0
-4	3

$x \leq 0$