# Foundations 20

## *Unit 5 – Oblique Triangle Trigonometry*

***F20.5 :*** *Demonstrate understanding of the cosine law and sine law*

***Adapted from Nelson Foundations of Mathematics 11***

Name: _________________
Date: __________________

# Key Terms

**Triangle –** A polygon with three sides and three angles.

**Acute –** An angle less than 90°

**Acute Triangle –** A triangle which has all acute angles (less than 90°).

**Trigonometric Ratios –** The sine, cosine and tangent ratio, used to find lengths and measures in a triangle.

**Sine Ratio –** The ratio of the opposite side and the hypotenuse. (SOH)

**Cosine Ratio** – The ratio of the adjacent side and the hypotenuse. (CAH)

**Tangent Ratio** – The ratio of the opposite side and the adjacent side. (TOA)

**Right Triangle –** A triangle that has one 90° angle.

**Sine Law –** in any acute triangle: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine Law** – In any acute triangle: $a^2 = b^2 + c^2 - 2bc \cos A$

**Measure –** How many degree's there are in an angle

**Oblique Triangle** – A triangle that does not contain a 90° angle.

**Supplementary** – Two or more angles that add up to 180°.

**Ambiguous Case** – A situation in which two triangles can be drawn, given the available information; the ambiguous case may occur when the given measurements are the lengths of two sides and the measure of an angle that is not contained by the two sides (SSA)

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# Unit 5 - Checklist

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- ☐ Lesson 1 – Portfolio Question

- ☐ Lesson 2 – Video
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- ☐ Portfolio – Post Questions
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- ☐ Portfolio – Unit Summary

- ☐ Test

# Lesson 1 – Primary Trigonometric Ratios of Obtuse Angles

**Oblique Triangle** – A triangle that does not contain a 90° angle.

**Supplementary** – Two or more angles that add up to 180°.

**Find the Supplement of an Angle**
1) Determine the measure of your original angle
2) Subtract that angle from 180°

**Example 1:** Fill out the following chart to four decimal places.

| $\theta$ | sin$\theta$ | cos$\theta$ | tan$\theta$ | (180° - $\theta$) | sin (180° - $\theta$) | cos (180° - $\theta$) | tan (180° - $\theta$) |
|---|---|---|---|---|---|---|---|
| 100° | | | | | | | |
| 110° | | | | | | | |
| 120° | | | | | | | |
| 130° | | | | | | | |

**For any angle *θ*:**

- sin$\theta$ = sin (180° - $\theta$)
- cos$\theta$ = - cos (180° - $\theta$)
- tan$\theta$ = - tan (180° - $\theta$)

**Determining if Angles are Supplementary**
1) Take the trigonometric ratio of each angle
2) If it is the Sine Ratio, determine if the numbers are equal
3) If it is the Cosine or Tangent Ratio, determine if the numbers are equal with the opposite sign

**Example 2:** Determine if the following equations are valid.

a) sin 20° = sin 130°

b) cos 50° = - cos 130°

c) tan 50° = tan 150°

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Lesson 1 – Primary Trigonometric Ratios of Obtuse Angles

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**Finding Angles with Equal or Opposite Trigonometric Ratios**

1) Determine your original angle and if your angle is acute or obtuse
2) Find the supplement of that angle

**Example 3:** Find an angle that will have equal or opposite trigonometric ratios for the following:

a) sin 20°　　b) cos 40°　　d) tan 146°

**Determining Angles from a Trigonometric Ratio**

1) Find the ratio as a decimal (if it is already a decimal this step is done)
2) Use $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ depending on which ratio you are using
3) Use the angle found to also find the supplementary angle
4) Check to see if the angles have equal or opposite trigonometric ratios

**Example 4:** Determine two angles from each of the following sine ratios

a) 0.4521　　b) $\frac{2}{3}$　　c) 0.11

# Lesson 2 – Sine Law with Obtuse Triangles

**Sine Law:** $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ or $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Using Sine Law to find the Measure of an Obtuse Angle**

1) Set up your ratios for the Sine Law with Angle on the top
2) Solve for your missing angle as you normally would
3) Find the supplement of your answer to find the equivalent obtuse angle

**Example 1:** In Triangle ABC; a = 65.0 cm, b = 40.0 cm, <B = 23°. Find the measure of angle A.

**Solving a Problem Using the Sine Law or the Primary Trigonometric Ratios**

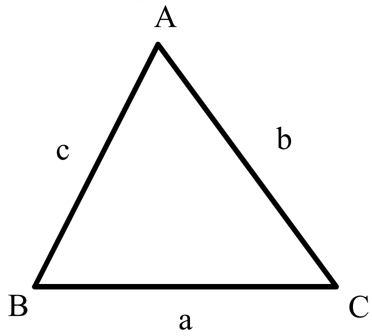
1) Determine if the triangle you are finding has a right angle
2) If it does NOT have a right angle (90°) use the Sine Law
3) If it is a right triangle use the primary trigonometric ratios

**Primary Trigonometric Ratios**: $sin\theta = \frac{opposite}{hypotenuse}$, $cos\theta = \frac{adjacent}{hypotenuse}$, $tan\theta = \frac{opposite}{adjacent}$

**Example 2:** A weather balloon is tethered to the ground. You observe the balloon above the ground at an inclination of 7° from 250 m away. Your friend is standing where the balloon is tethered and observes the angle of elevation to be 82°. Find the height that the balloon is above the ground.

# Lesson 3 – The Cosine Law with Obtuse Triangles

**Cosine Law** – In any acute triangle, and can solve problems where the Sine Law does not apply.




$c^2 = a^2 + b^2 - 2ab\mathit{Cos}C$

$a^2 = b^2 + c^2 - 2bc\mathit{Cos}A$

$b^2 = a^2 + c^2 - 2ac\mathit{Cos}B$

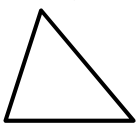
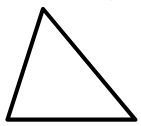
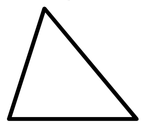
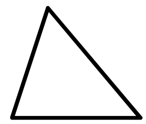
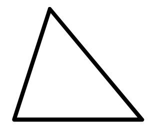
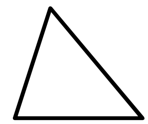
**Using the Cosine Law with an Obtuse Triangle**

1) The Cosine Law holds for obtuse triangles
2) Apply the cosine law as you would an acute triangle

**Example 1:** A roof of a house consists of two slanted sections. The two slants are 17.0 ft and 20.3 ft. The width of the roof is 33.5 ft. Determine the angle at the top of the roof.

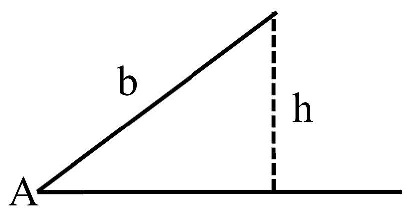
# Lesson 4 – Determining Number of Triangles

**The Ambiguous Case** – A situation in which two triangles can be drawn, given the available information

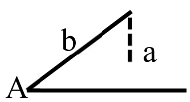
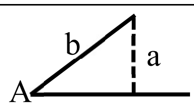
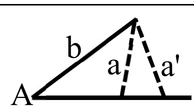
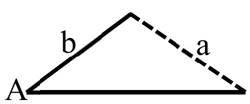
| When an Ambiguous Case Might Occur |
|---|
| - When you are given two lengths and an angle that is not contained **(SSA)**       |

**Determining the Number of Possible Triangles**

1) Sketch the triangle
2) Let a = the side opposite the angle
3) Let b = the side adjacent to the angle
4) Using side b and the sine ratio find h
5) Compare your values of a, b and h




$$\sin A = \frac{h}{b}$$

| Triangle | Compare a, b and h | Number of possible triangles |
|---|---|---|
|   | a < h | No Possible Triangles |
|   | a = h | One Possible Triangle (Right Triangle) |
|   | a < b and a > h | Two possible Triangles |
|   | a > b | One Possible Triangle |

**Example 1:** Given each SSA situation for Δ ABC, determine how many triangles are possible.

a) <A = 30°, a = 4 m, b = 12 m

b) <A = 30°, a = 6 m , b = 12 m

c) <A = 30°, a = 8 m, b = 12 m

d) <A = 30°, a = 15 m , b = 12 m

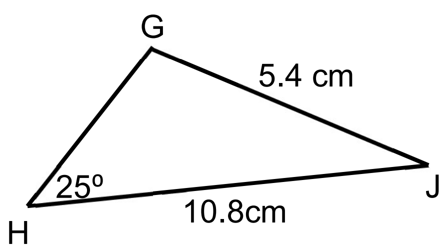
# Lesson 5 – The Ambiguous Case

**The Ambiguous Case** – A situation in which two triangles can be drawn, given the available information

**Solving a Problem With The Ambiguous Case**

1) Check to see if your triangle is SSA and a possible ambiguous case
2) Determine the possible number of triangles
3) Solve for your missing angle
4) Find the supplementary angle to the angle you have found
5) Set up two possible options: One acute triangle and one obtuse triangle
6) Solve for the remaining angle and side

**Example 1:** In $\Delta HGJ$, determine the measure of $\angle$ G to the nearest degree.



**Example 2:** Martina and Carl are part of a team that is studying weather patterns. The team is about to launch a weather balloon to collect data. Martina's rope is 7.8 m long and makes an angle of 36° with the ground. Carl's rope is 5.9 m long. Assuming that Martina and Carl form a triangle in a vertical plane with the weather balloon, what is the distance between Martina and Carl, to the nearest tenth of a metre?

# Lesson 5 – Worksheet

1. Solve the following triangle(s) based on the given information: $<A = 74$, $a = 59.2$, and $c = 60.3$

2. Solve the following triangle(s) based on the given information: $<A = 35°$, $a = 10$, and $b = 13$.

3. Solve the following triangle(s) based on the given information: $<A = 30°$, side $b= 12$, and side $a = 8$

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Lesson 5 – The Ambiguous Case

4. Solve the following triangle(s) based on the given information: $AC = 13$, $BC = 8$, and $\angle A = 36°$

5. Solve the following triangle(s) based on the given information: $\angle A = 40°$, $a = 5$, and $b = 6$

6. Solve the following triangle(s) based on the given information: $\angle A = 62°$ , $a = 2.8$, $b = 3.0$

# Lesson 6 – Solving Problems with Obtuse Triangles

**Solving Problems with Obtuse Triangles**

1) Read the question all the way through, circle or underline important aspects
2) Draw a diagram
3) Use the Sine Law, Cosine Law, or Primary Trigonometric Ratios to solve the problem.

**Example 1:** A surveyor is in a helicopter and trying to find the width of a lake. The helicopter is 1610 m above the forest and notices the angle of depression to opposite shores of the lakes to be 45° and 82°. The helicopter and the two points are in the same vertical plane. Determine the width of the lake.

**Example 2:** A communication tower is 35 m tall. David is due north of the tower and measures the angle of elevation of the top of the tower as 70°. Brenda is on a bearing of 175° from the tower and measures the angle of elevation of the top of the tower as 50°. To the nearest metre, determine the distance between Brenda and David.